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Quark-resonance model*

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Abstract. We construct an effective Lagrangian for low energy hadronic interactions through an infinite expansion in inverse powers of the low energy cutoff Λ_χ of all possible chiral invariant non-renormalizable interactions between quarks and mesons degrees of freedom arising from the bosonization of a general Nambu-Jona Lasinio type Lagrangian including all multiquark effective interactions. We restrict our analysis to the leading terms in the $1/N_c$ expansion and to the divergent part of the resonance effective Lagrangian resulting from the integration over the quark degrees of freedom. Indeed, the effective expansion is in $(Q^2/\Lambda_\chi^2)^P \ln(\Lambda_\chi^2/Q^2)^M$ and we show that, while the finite terms cannot be traced back to a finite number of non renormalizable interactions, the divergent ones of order $(Q^2/\Lambda_\chi^2)\ln(\Lambda_\chi^2/Q^2)$ receive contributions from a finite set of $1/\Lambda_\chi^2$ terms of the original quark-meson Lagrangian. These terms modify the behaviour of physical quantities in the intermediate Q^2 region. We explicitely discuss their relevance for the two point vector currents Green's function.

1 Introduction

Effective chiral Lagrangians have become a relatively powerful technique to describe hadronic interactions at low energy, i.e. below the chiral symmetry breaking scale $\Lambda_\chi \simeq 4\pi f_\pi \sim 1$ GeV. Chiral perturbation theory (ChPt) [1, 2] describes the low energy interactions among the pseudoscalar mesons π, K, η, which are the lightest asymptotic states of the hadron spectrum and are identified with the Goldstone bosons of the broken chiral symmetry. The inclusion of resonance degrees of freedom in the model (vectors, axials, scalars, pseudoscalars and flavour singlets scalar and pseudoscalar) allows to describe the interactions of all the particles below Λ_χ [3–7]. This approach has a disadvantage connected with the non renormalizability of the effective low energy theory. The chiral expansion (i.e. the expansion in powers of derivatives of the low energy fundamental fields) results as an infinite sum over chiral invariant operators of increasing dimensionality. At each order in the chiral expansion the number of terms increases and the theory looses predictivity at higher orders. Many attempts have been done to reformulate the model in a more predictive fashion, both in the non anomalous [6] and in the anomalous sector [7] of the theory.

In particular, there have been attempts to derive the low energy effective theory from the more fundamental theory which describes the interactions of quarks and gluons. The first attempt to connect the low energy effective theory of pseudoscalar mesons and resonances with QCD has been proposed in [8], where an application to strong interactions of the old and well known Nambu-Jona Lasinio (NJL) model [9–11] is made. The QCD Lagrangian is modified at long distances (i.e. below the cutoff Λ_χ) by an effective 4-quarks interaction Lagrangian which remains chirally invariant.

The resonance and pseudoscalar mesons fields are introduced in the model through the bosonization of the fermion effective action.

The ENJL model proposed in [8] includes only interaction terms which are leading in an expansion in inverse powers of the cutoff Λ_χ. This is a reasonable approximation when we are interested in the behaviour of the effective theory for light mesons at a very low energy. Higher order terms bring powers of the derivative expansion term ∂/Λ_χ which are indeed suppressed.
This is not the case in the intermediate and high energy region, i.e. throughout the resonance region, where non renormalizable power-like corrections arising from higher order terms can be dominant. The ENJL is not the full answer in the intermediate Q^2 region, while it can be satisfactorily used to derive the effective Lagrangian of the pseudo-Goldstone bosons (pions) at $Q^2 = 0$.

The presence of next-to-leading terms in the ENJL formulation, i.e. higher dimensional operators with four or more fermion fields, leads after bosonization to an effective quark-resonance Lagrangian whose logarithmically divergent part contains both renormalizable and non renormalizable terms, i.e. next-to-leading contributions at low energy.

In Sect. 2 we construct the quark-resonance model: we review the derivation of the leading terms from the ENJL

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model and we discuss with specific examples how next-to-leading terms in the quark-resonance model can be traced back to next-to-leading terms in the original ENJL model. In Sect. 3 we discuss the general parametrizations of those terms in the quark-resonance model which give next-to-leading contributions to the final effective meson-resonance Lagrangian. In Sect. 4 we specialize to the vector part of the effective meson-resonance Lagrangian and we study the running of the parameters of the leading ENJL Lagrangian induced by the next-to-leading corrections. In Sect. 5 we concentrate on the case of the two-point vector correlation function, where we are able to extract significative informations on the Q^2 behaviour of the real part of the invariant functions from the existing data on the total e^+e^- hadron cross section in the $I = 1$ channel. The results can be directly compared with the predictions obtained in the ENJL framework [12, 13]. The corrections are shown to improve the agreement with the experimental data.

2 The model

The effective quark models describing low energy strong interactions assume that the result of integrating over high frequency modes in the original QCD Lagrangian, defined above a given energy cutoff, can be expressed by additional non-renormalizable interactions.

For strong interactions the natural cutoff is the scale at which chiral symmetry spontaneously breaks: $\Lambda_\chi \simeq 1$ GeV. The cutoff sets the limit below which only the "low frequency modes" of the theory are excited.
The QCD Lagrangian for the low frequency modes is modified as follows:

$$\mathscr{L}_{QCD} \rightarrow \mathscr{L}_{QCD}^{\Lambda_\chi} + \mathscr{L}_{N.R.}(n-fermion). \tag{1}$$

$\mathscr{L}_{QCD}^{\Lambda_\chi}$ is the standard QCD Lagrangian where only the low-frequency modes of quarks and gluons are present:

$$\mathscr{L}_{QCD}^{\Lambda_\chi} = \bar{q}(i\hat{D} - m_0)q, \tag{2}$$

with $D_\mu = \partial_\mu + iG_\mu$. The *current* quarks $q_{L,R}$ transform as $q_{L,R} \rightarrow g_{L,R}q_{L,R}$ under the chiral flavour group $SU(3)_L \times SU(3)_R$, with elements $g_{L,R}$. The QCD Lagrangian (2) with zero quark masses ($m_0 = 0$) is invariant under global chiral transformations. The low energy Green's functions generating functional in presence of external sources v, a, s, p is associated to the modified low energy QCD Lagrangian:

$$\begin{aligned}\mathscr{L}_{QCD}^{\Lambda_\chi} = \bar{q}(i\hat{D} - m_0)q + \bar{q}\gamma_\mu(v_\mu + \gamma_5 a_\mu)q \\ -\bar{q}(s - i\gamma_5 p)q.\end{aligned} \tag{3}$$

The vector-like sources $v = (r+l)/2, a = (r-l)/2$ transform under local chiral transormations as

$$\begin{aligned}l_\mu &\rightarrow g_L l_\mu g_L^\dagger - ig_L^\dagger \partial_\mu g_L \\ r_\mu &\rightarrow g_R r_\mu g_R^\dagger - ig_R^\dagger \partial_\mu g_R.\end{aligned} \tag{4}$$

and turn derivatives into covariant derivatives. The QCD Lagrangian (3) with zero quark masses ($m_0 = 0$) becomes locally chiral invariant.

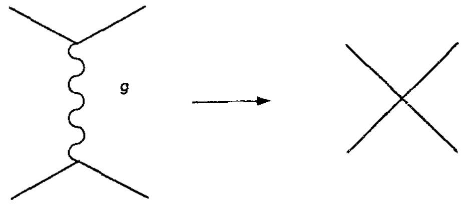

Fig. 1. The QCD diagram with one gluon exchange generates an effective 4-quark interaction vertex

The second term in (1) is the most general non-renormalizable set of higher dimensional local n-fermion interactions which respect the symmetries of the original theory and are suppressed at low energy by powers of Q^2/Λ_χ^2.

Recently, the Nambu- Jona Lasinio (NJL) model has been reanalyzed in a systematic way in the framework of hadronic low energy interactions [8]. Many applications and reformulations can be found in [11].
The extended version of the NJL model (ENJL) includes in $\mathscr{L}_{N.R.}(n-fermion)$ all lowest dimension operators: 4-fermion local interactions which are leading in the $1/N_c$ expansion [14] (colour singlets) and respect all the symmetries of the original theory (chiral symmetry, Lorentz invariance, P and C invariance). The form of the effective Lagrangian is then uniquely determined:

$$\mathscr{L}^{\mathrm{ENJL}} = \mathscr{L}_{\mathrm{QCD}}^{\Lambda_\chi} + \mathscr{L}_{\mathrm{NJL}}^{S,P} + \mathscr{L}_{\mathrm{NJL}}^{V,A}, \tag{5}$$

with

$$\mathscr{L}_{\mathrm{NJL}}^{S,P} = \frac{8\pi^2 G_S(\Lambda_\chi)}{N_c \Lambda_\chi^2} \sum_{a,b} (\bar{q}_R^a q_L^b)(\bar{q}_L^b q_R^a) \tag{6}$$

and

$$\begin{aligned}\mathscr{L}_{\mathrm{NJL}}^{V,A} = -\frac{8\pi^2 G_V(\Lambda_\chi)}{N_c \Lambda_\chi^2} \\ \sum_{a,b}\left[(\bar{q}_L^a \gamma_\mu q_L^b)(\bar{q}_L^b \gamma^\mu q_L^a) + (L \rightarrow R)\right].\end{aligned} \tag{7}$$

As pointed out in [8] the 4-quark effective vertex can be thought of as a remnant of a "low frequency" one gluon exchange (see Fig. 1). The gluon propagator modified at high energy with a cutoff

$$\frac{1}{Q^2} \rightarrow \int_0^{\frac{1}{\Lambda_\chi^2}} d\tau e^{-\tau Q^2} \tag{8}$$

leads to a local effective 4-quark interaction

$$\frac{g_s^2}{\Lambda_\chi^2}\left(\bar{q}\gamma_\mu \frac{\lambda^{(a)}}{2} q\right)\left(\bar{q}\gamma_\mu \frac{\lambda_{(a)}}{2} q\right). \tag{9}$$

By means of the Fierz-identities one gets the S, P, V, A combinations of (6,7) with the identification $G_S = 4G_V$.

The non-renormalizable part of the fermion action $S_{NR}(q)$ can be represented in terms of *auxiliary boson fields* as:

$$e^{iS_{NR}[q]} = \int \mathscr{D}B e^{iS[B,q]}. \tag{10}$$

The previous relation introduces the meson degrees of freedom into the effective quark Lagrangian. The following two identities hold:

$$\exp i\int d^4x\mathscr{L}_{S,P}(x)=\int\mathscr{D}H\exp i\int d^4x$$
$$\left\{-(\bar q_L H^\dagger q_R+h.c.)-\frac{N_c\Lambda_\chi^2}{8\pi^2G_S}tr(HH^\dagger)\right\}$$
$$\exp i\int d^4x\mathscr{L}_{V,A}(x)=\int\mathscr{D}L_\mu\mathscr{D}R_\mu\exp i\int d^4x$$
$$\left\{\bar q_L\gamma_\mu L^\mu q_L\right.$$
$$\left.+\frac{N_c\Lambda_\chi^2}{8\pi^2G_V}\frac{1}{4}\mathrm{tr}(L_\mu L^\mu)+(L\to R)\right\},\tag{11}$$

where we have introduced three auxiliary fields: a scalar field $H(x)$ and the right-handed and left-handed fields L_μ and R_μ. Under the chiral group they transform as:

$$H\to g_R H g_L^\dagger$$
$$L_\mu\to g_L L_\mu g_L^\dagger$$
$$R_\mu\to g_R R_\mu g_R^\dagger.\tag{12}$$

The field H can be decomposed into the product of a new scalar field M times a unitary field U:

$$H=MU=\xi\tilde H\xi,\tag{13}$$

where the field ξ is the square root of the field U: $\xi^2=U$. The physical fields are obtained by redefining the auxiliary fields as follows:

$$H=\xi\tilde H\xi$$
$$W_\mu^+=\xi L_\mu\xi^\dagger+\xi^\dagger R_\mu\xi$$
$$W_\mu^-=\xi L_\mu\xi^\dagger-\xi^\dagger R_\mu\xi\ .\tag{14}$$

The new set of fields transforms homogeneously under chiral transformation:

$$\{\tilde H,W_\mu^+,W_\mu^-\}\to h\{\tilde H,W_\mu^+,W_\mu^-\}h^\dagger,\tag{15}$$

where h is a non linear representation of the chiral group.

We redefine also the fermion fields by replacing the *current* quarks $q_{L,R}$ with the *constituent* quarks:

$$Q_L=\xi q_L\quad Q_R=\xi^\dagger q_R$$
$$\bar Q_L=\bar q_L\xi^\dagger\quad \bar Q_R=\bar q_R\xi.\tag{16}$$

They transform under the chiral group $G=SU(3)_L\times SU(3)_R$ as:

$$Q_L\to h(\Phi,g_L,g_R)Q_L\quad Q_R\to h(\Phi,g_L,g_R)Q_R,\tag{17}$$

where the matrix $h(\Phi,g_L,g_R)$ acts on the element ξ of the coset group $G/SU(3)_V$

$$\xi(\Phi)\to g_R\xi(\Phi)h^\dagger=h\xi(\Phi)g_L^\dagger.\tag{18}$$

The quark field Q is defined as $Q=Q_L+Q_R$.

In terms of the new variables the euclidean generating functional of the ENJL model reads:

$$Z[v,a,s,p]=\int\mathscr{D}\xi\mathscr{D}\tilde H\mathscr{D}L_\mu\mathscr{D}R_\mu e^{-\Gamma_{\mathrm{eff}}[\xi,W_\mu^+,W_\mu^-,\tilde H;v,a,s,p]}$$
$$e^{-\Gamma_{\mathrm{eff}}[\xi,W_\mu^+,W_\mu^-,\tilde H;v,a,s,p]}=$$
$$\exp\Big(-\int d^4x\Big\{\frac{N_c\Lambda_\chi^2}{8\pi^2G_S(\Lambda_\chi)}tr\tilde H^2$$
$$+\frac{N_c\Lambda_\chi^2}{16\pi^2G_V(\Lambda_\chi)}\frac{1}{4}tr(W_\mu^+W^{+\mu}+W_\mu^-W^{-\mu})\Big\}\Big)\times$$
$$\frac{1}{Z}\int\mathscr{D}G_\mu\exp\Big(-\int d^4x\frac{1}{4}G_{\mu\nu}^{(a)}G^{(a)\mu\nu}\Big)$$
$$\int\mathscr{D}Q\mathscr{D}\bar Q\exp\Big(\int d^4x\bar QD_EQ\Big),\tag{19}$$

where we have defined the total differential operator D_E as follows:

$$D_E=\gamma_\mu\mathscr{D}_\mu-\frac{1}{2}(\Sigma-\gamma_5\Delta)-\tilde H(x),\tag{20}$$

with the covariant derivative acting on the chiral quark field given by:

$$\mathscr{D}_\mu=\partial_\mu+iG_\mu+\Gamma_\mu-\frac{i}{2}W_\mu^+-\frac{i}{2}\gamma_5(\xi_\mu-W_\mu^-).\tag{21}$$

The field Γ_μ acts like a vector field and is defined by:

$$\Gamma_\mu=\frac{1}{2}\{\xi^\dagger d_\mu\xi+\xi d_\mu\xi^\dagger\}=\frac{1}{2}\{\xi^\dagger[\partial_\mu-i(v_\mu+a_\mu)]\xi$$
$$+\xi[\partial_\mu-i(v_\mu-a_\mu)]\xi^\dagger\}.\tag{22}$$

It transforms inhomogeneously under the local vector part of the chiral group

$$\Gamma_\mu\to h\Gamma_\mu h^\dagger+h\partial_\mu h^\dagger\tag{23}$$

and makes the derivative on the Q field invariant under local vector transformations.
The field ξ_μ is like an axial current and is defined by:

$$\xi_\mu=i\{\xi^\dagger d_\mu\xi-\xi d_\mu\xi^\dagger\}$$
$$=i\{\xi^\dagger[\partial_\mu-i(v_\mu+a_\mu)]\xi-\xi[\partial_\mu-i(v_\mu-a_\mu)]\xi^\dagger\}$$
$$=\xi_\mu^\dagger.\tag{24}$$

It transforms homogeneously under the chiral group G:

$$\xi_\mu\to h\xi_\mu h^\dagger.\tag{25}$$

The field strenghts of Γ_μ and ξ_μ fields are given by:

$$\Gamma_{\mu\nu}=\partial_\mu\Gamma_\nu-\partial_\nu\Gamma_\mu+[\Gamma_\mu,\Gamma_\nu]$$
$$\xi_{\mu\nu}=d_\mu\xi_\nu-d_\nu\xi_\mu=\partial_\mu\xi_\nu+[\Gamma_\mu,\xi_\nu]-(\mu\leftrightarrow\nu),\tag{26}$$

where the covariant derivative d_μ of the ξ_μ field has been introduced and both transform homogeneously under the chiral group. They are related to the field strenghts

$$f_{\mu\nu}^{\pm}=\xi F_{\mu\nu}^L\xi^\dagger\pm\xi^\dagger F_{\mu\nu}^R\xi\tag{27}$$

through the identities

$$\Gamma_{\mu\nu}=-\frac{i}{2}f_{\mu\nu}^++\frac{1}{4}[\xi_\mu,\xi_\nu]$$
$$\xi_{\mu\nu}=f_{\mu\nu}^-.\tag{28}$$

The fields Σ and Δ are defined by:

$$\Sigma=\xi^\dagger\mathscr{M}\xi^\dagger+\xi\mathscr{M}\xi$$
$$\Delta=\xi^\dagger\mathscr{M}\xi^\dagger-\xi\mathscr{M}\xi.\tag{29}$$

They are both proportional to the quark mass matrix $\mathscr{M}$ and vanish in the chiral limit. The field $\tilde H(x)$ is the auxiliary scalar field of the bosonized action and can be parametrised as

$$\tilde{H}(x) = M_Q \mathbf{1} + \sigma(x), \tag{30}$$

where we have split the $\tilde{H}$ field into its vacuum expectation value and the fluctuation around it. The quantity M_Q is the value of the $\tilde{H}(x)$ field (used in the so called mean field approximation of the ENJL model) which minimizes the effective action in absence of other external fields:

$$\frac{\delta \Gamma_{\mathrm{eff}}(\tilde{H},..)}{\delta \tilde{H}}\Big|_{\xi=1, W_\mu^+=W_\mu^-=0; v,a,s,p=0; \tilde{H}=<\tilde{H}>} = 0. \tag{31}$$

$M_Q \neq 0$ corresponds to broken chiral symmetry [15]. Its value is the solution of the mass gap equation generated by (31).

In the leading effective action (19) two constants appear: the scalar coupling G_S and the vector coupling G_V. They are functions of the cutoff Λ_χ and their estimate involves non-perturbative contributions.

The fundamental fields of the bosonized action of the *constituent* quarks are $W_\mu^+, W_\mu^-, \tilde{H}$. They have the usual chiral properties of the physical low energy meson fields. The field ξ appears as a consequence of the transition from *current* to *constituent* quarks.
A full effective quark model *à la NJL* contains a priori an infinite tower of n-fermion operators with increasing dimensionality: the ENJL 4-fermion interactions are the leading terms both in $1/\Lambda_\chi$ and $1/N_c$ expansions.

The Quark-Resonance (QR) model is the bosonization of the full effective *current* quark model *à la NJL.*

The resulting quark-resonance Lagrangian is a non-renormalizable Lagrangian which contains all possible interaction terms between quarks and resonances. Physical meson fields are introduced by the transformation from the *current* quark base to the *constituent* quark base defined in (16). This implies that the equivalence between the most general chirally invariant *current* quark-resonance Lagrangian and the most general chirally invariant *constituent* quark-resonance Lagrangian holds with two *caveats*:

i) The presence of ξ_μ and Γ_μ currents defined by (22) and (24) in the *constituent* quark Lagrangian is entirely due to the transformation from *current* to *constituent* quarks of (16). The following identities hold:

$$\begin{aligned} &\overrightarrow{\nabla}_\mu\, Q_L = \xi\, \overrightarrow{d}_\mu\, q_L \quad \overrightarrow{\nabla}_\mu\, Q_R = \xi^\dagger\, \overrightarrow{d}_\mu\, q_R \\ &\bar{Q}_L\, \overleftarrow{\nabla}_\mu^{C^T} = \bar{q}_L\, \overleftarrow{d}_\mu\, \xi^\dagger \quad \bar{Q}_R\, \overleftarrow{\nabla}_\mu^{C^T} = \bar{q}_R\, \overleftarrow{d}_\mu\, \xi, \end{aligned} \tag{32}$$

where d_μ is the covariant derivative of the *current* quark field $d_\mu q_{L,(R)} = \partial_\mu q_{L,(R)} - i l(r)_\mu q_{L,(R)}$, $\overrightarrow{\nabla}_\mu$ is the covariant derivative defined in (21):

$$\overrightarrow{\nabla}_\mu \equiv \overrightarrow{\partial}_\mu + \Gamma_\mu - \frac{i}{2}\gamma_5 \xi_\mu, \tag{33}$$

which acts on the constituent quark Q and $\overleftarrow{\nabla}_\mu^{C^T}$ is its charge conjugate

$$\overleftarrow{\nabla}_\mu^{C^T} \equiv \overleftarrow{\partial}_\mu - \Gamma_\mu - \frac{i}{2}\gamma_5 \xi_\mu, \tag{34}$$

which acts on the constituent anti-quark $\bar{Q}$. ξ_μ and Γ_μ currents can only appear in the combinations (33), (34) through the covariant derivatives on constituent quarks.

ii) The vector field W_μ^+ and the axial-vector field W_μ^- can only appear in the combination $W_\mu^+ - \gamma_5 W_\mu^-$ and its charge conjugate, i.e. in the combination of the leading ENJL Lagrangian. For example, at the leading order a term $\bar{Q}\gamma_\mu W_\mu^+ Q$ in the constituent quark base, which would respect chiral invariance, leads to the term

$$\begin{aligned} &\bar{q}_L \gamma_\mu L_\mu q_L + \bar{q}_R \gamma_\mu R_\mu q_R + \bar{q}_L \gamma_\mu (\xi^\dagger)^2 R_\mu \xi^2 q_L + \\ &\bar{q}_R \gamma_\mu \xi^2 L_\mu (\xi^\dagger)^2 q_R, \end{aligned} \tag{35}$$

where the last two terms contain powers of the pseudoscalar ξ field trapped in between and are absent in the *current* quark base. They are not present in the combination $W_\mu^+ - \gamma_5 W_\mu^-$.
The QCD euclidean generating functional of the correlation functions at low energy within the Quark-Resonance model is given by:

$$\begin{aligned} Z[v,a,s,p] &= e^{W[v,a,s,p]} \\ &= \int \mathscr{D}R\, e^{-\Gamma_{\mathrm{eff}}[R;v,a,s,p]}, \end{aligned} \tag{36}$$

where R contains the set of fields introduced by the bosonization of the low energy QCD effective Lagrangian and the effective action Γ_{eff} is given by

$$\begin{aligned} &e^{-\Gamma_{\mathrm{eff}}[R;v,a,s,p]} = \frac{1}{Z}\int \mathscr{D}G_\mu \exp\Big(-\int d^4x \frac{1}{4} G_{\mu\nu}^{(a)} G^{(a)\mu\nu}\Big) \\ &e^{-f[R]} \int \mathscr{D}Q\mathscr{D}\bar{Q} \exp\Big[\int d^4x \\ &\Big(\bar{Q}\gamma^\mu(\partial_\mu + iG_\mu)Q + \sum_0^\infty \Big(\frac{1}{\Lambda_\chi}\Big)^n \bar{Q}RQ\Big)\Big], \end{aligned} \tag{37}$$

where the functional $f[R]$ in (37) contains the terms with auxiliary boson fields which are not coupled to fermions.

The most general structure of the R operator can be represented by:

$$R = \beta(\Lambda_\chi) \times \{\gamma_{\mathrm{Dirac}}\} \times \{W_\mu^+, W_\mu^-, \tilde{H}\} \times \{\nabla_\mu^n, (\nabla_\mu^{C^T})^n\}, \tag{38}$$

where the couplings $\beta(\Lambda_\chi)$ are not deducible from symmetry principles. ∇_μ and $\nabla_\mu^{C^T}$ are defined in (33) and (34) and the set $\{W_\mu^+, W_\mu^-, \tilde{H}\}$ contains all possible fields introduced by the bosonization which can couple to the quark bilinears and which can be identified with the physical degrees of freedom of the low energy effective theory; $W_\mu^\pm$ fields appear in the combinations $W_\mu^+ \pm \gamma_5 W_\mu^-$ and the pseudoscalar mesons are hidden in the covariant derivatives $\nabla_\mu, \nabla_\mu^{C^T}$. As it is shown in detail in [8], the integration over quark fields induces a mixing between the axial field W_μ^- and the axial current ξ_μ which is leading in the chiral expansion: a diagonalization of the final meson effective action is required to define the true physical axial and pseudoscalar meson fields.

The QR Lagrangian at leading order in the $1/\Lambda_\chi$ expansion and in the $1/N_c$ expansion, in the *constituent* quark base, coincides with the bosonization of the ENJL model of (19). The additional quark-resonance interaction terms originate from the bosonization of non-renormalizable n-quark (n$\geq$ 4) vertices, with the insertion of powers of the differential operator d^2/Λ_χ^2 and with the covariant derivative d involving external sources. The QR Lagrangian so defined is locally chiral invariant. At leading order in the $1/N_c$ expansion it can be constructed from the locally chiral invariant building blocks

(1) $\bar{q}\hat{d}q$

(2) $\frac{1}{\Lambda_\chi^2}(\bar{q}_L^a q_R^b)(\bar{q}_R^b q_L^a)$

(3) $\frac{1}{\Lambda_\chi^2}[(\bar{q}_L^a\gamma_\mu q_L^b)(\bar{q}_L^b\gamma_\mu q_L^a) + (L\to R)],$ (39)

with the insertion of powers of d^2/Λ_χ^2 acting on quarks. a, b are flavour indices and the bilinears in parenthesis are colour singlets.

In addition there can be multifermion operators which are products of flavour singlet blocks which will appear in a quark-flavour singlet resonance Lagrangian. They are also suppressed in the $1/N_c$ expansion. We restrict our analysis to the flavour non singlet resonances. Terms with $\sigma_{\mu\nu}$ are proportional to the bare quark mass term which is set to zero in this analysis.

Summarizing, bosonization of leading N_c terms requires meson fields which are flavour octets with scalar, pseudoscalar, vector and axial-vector quantum numbers.

We give a couple of examples of how the bosonization of the multiquark terms builds the *constituent* quark-resonance Lagrangian.

i) Terms with four quarks with the insertion of derivatives. We consider the term:

$$O_4 = \frac{1}{\Lambda_\chi^2}\left[\bar{q}_L\gamma^\mu q_L\left(\bar{q}_L\gamma_\mu\frac{\overrightarrow{d}^2}{\Lambda_\chi^2}q_L + \bar{q}_L\frac{\overleftarrow{d}^{C^T 2}}{\Lambda_\chi^2}\gamma_\mu q_L\right)\right.$$
$$\left. +(L\to R)\right], \quad (40)$$

where we have explicitely written the charge conjugated derivative term which makes the whole expression C invariant.

The bosonization of the operator above together with the leading four quarks vector-like term leads to:

$$e^{i\int d^4x\mathscr{L}_V} = \int\int DL_\mu DR_\mu \exp\Big(i\int d^4x$$
$$\left\{\frac{N_c\Lambda_\chi^2}{8\pi^2 G_V}\frac{1}{4}trL_\mu^2 + \bar{q}_L\gamma^\mu L^\mu q_L + \beta\bar{q}_L\gamma_\mu\left\{L^\mu, \frac{\overrightarrow{d}^2}{\Lambda_\chi^2}\right\}q_L\right.$$
$$\left.+(L\to R)\right\}\Big), \quad (41)$$

with $\mathscr{L}_V$ given by:

$$\mathscr{L}_V = -\frac{8\pi^2 G_V}{N_c\Lambda_\chi^2}\left\{\left[\bar{q}_L\gamma^\mu q_L + \right.\right.$$
$$\left.\left.\beta\left(\bar{q}_L\gamma_\mu\frac{\overrightarrow{d}^2}{\Lambda_\chi^2}q_L + \bar{q}_L\frac{\overleftarrow{d}^{C^T 2}}{\Lambda_\chi^2}\gamma_\mu q_L\right)\right]^2 + (L\to R)\right\}. \quad (42)$$

In the *constituent* quark base the bosonized action can be easily rewritten as:

$$e^{i\int d^4x\mathscr{L}_V} = \int\int D\tilde{L}_\mu D\tilde{R}_\mu$$
$$\exp\Big(i\int d^4x\Big\{\frac{N_c\Lambda_\chi^2}{8\pi^2 G_V}\frac{1}{4}\mathrm{tr}\tilde{L}_\mu^2 + \bar{Q}_L\gamma^\mu\tilde{L}^\mu Q_L$$
$$+\beta\bar{Q}_L\gamma_\mu\{\tilde{L}^\mu, \frac{\overrightarrow{\nabla}^2}{\Lambda_\chi^2}\}Q_L + \tilde{L}\to\tilde{R}\Big\}\Big)$$
$$= \int\int DW_\mu^+ DW_\mu^- \exp\Big(i\int d^4x$$
$$\Big\{\frac{N_c\Lambda_\chi^2}{8\pi^2 G_V}\frac{1}{8}\mathrm{tr}(W_\mu^{+2} + W_\mu^{-2}) + \bar{Q}\gamma^\mu(W_\mu^+ - \gamma_5 W_\mu^-)Q$$
$$+\beta\bar{Q}\gamma_\mu\{W_\mu^+ - \gamma_5 W_\mu^-, \frac{\overrightarrow{\nabla}^2}{\Lambda_\chi^2}\}Q\Big\}\Big), \quad (43)$$

where $\tilde{L}_\mu = \xi L_\mu\xi^\dagger$, $\tilde{R}_\mu = \xi^\dagger R_\mu\xi$ and the vector and axial fields $W_\mu^\pm$ have been defined in (14). The β term obtained is the term 2. of the vector set in the list (50) which will be introduced later on.

ii) Terms with six quarks. We consider a six-fermion interaction in the *current* quark base

$$O_6 = \frac{G_M}{\Lambda_\chi^6}\bar{q}\gamma_\mu q\Big[\bar{q}_R\overrightarrow{d}_\mu q_L\bar{q}_L q_R - \bar{q}_L q_R\bar{q}_R\overleftarrow{d}_\mu q_L$$
$$+\bar{q}_L\overrightarrow{d}_\mu q_R\bar{q}_R q_L - \bar{q}_R q_L\bar{q}_L\overleftarrow{d}_\mu q_R\Big], \quad (44)$$

with the derivative acting on the neighbouring field only. The form is constrained by invariance under P and C transformations. The Lagrangian which includes the leading four-fermion operators and the six-fermion operator (44) reads:

$$\mathscr{L}_{V,S} = -\frac{8\pi^2 G_V}{N_c\Lambda_\chi^2}\left[(\bar{q}_L\gamma^\mu q_L)^2 + (L\to R)\right]$$
$$+\frac{8\pi^2 G_S}{N_c\Lambda_\chi^2}\bar{q}_L q_R\bar{q}_R q_L + O_6, \quad (45)$$

i.e. its bosonization can be performed in two steps and leads to the introduction of both scalar and vector resonances. The first step introduces a scalar field:

$$e^{i\int d^4x\mathscr{L}_{V,S}} = \exp i\int d^4x$$
$$\left(-\frac{8\pi^2 G_V}{N_c\Lambda_\chi^2}[(\bar{q}_L\gamma^\mu q_L)^2 + (L\to R)]\right)\cdot$$
$$\int DM\exp\Big(i\int d^4x$$
$$\Big\{-\frac{N_c\Lambda_\chi^2}{8\pi^2 G_S}tr(M^\dagger M) - (\bar{q}_R M q_L + \bar{q}_L M^\dagger q_R)$$
$$-\frac{N_c G_M}{8\pi^2 G_S\Lambda_\chi^4}\bar{q}\gamma_\mu q(\bar{q}_R M\overrightarrow{d}_\mu q_L - \bar{q}_R\overleftarrow{d}_\mu M q_L +$$
$$\bar{q}_L M^\dagger\overrightarrow{d}_\mu q_R - \bar{q}_L\overleftarrow{d}_\mu M^\dagger q_R)\Big\}, \quad (46)$$

where the equality holds up to fermion terms of order $1/\Lambda_\chi^{10}$. The second step introduces the left and right-handed fields:

$$e^{i\int d^4x\mathscr{L}_{V,S}} = \int DMDL_\mu DR_\mu$$
$$\exp\Big(i\int d^4x\Big\{-\frac{N_c\Lambda_\chi^2}{8\pi^2 G_S}tr(MM^\dagger)$$

$$+\frac{N_c \Lambda_\chi^2}{8\pi^2 G_V}\frac{1}{4}tr(L_\mu^2 + R_\mu^2) - (\bar{q}_R M q_L + \bar{q}_L M^\dagger q_R)$$
$$+\bar{q}_L\gamma_\mu L_\mu q_L + \bar{q}_R\gamma_\mu R_\mu q_R + \frac{1}{2}\left(\frac{N_c}{8\pi^2}\right)^2 \frac{G_M}{G_S G_V}\frac{1}{\Lambda_\chi^2}$$
$$\left[\bar{q}_R\left(M L_\mu \overrightarrow{d}_\mu - \overleftarrow{d}_\mu M L_\mu + R_\mu M \overrightarrow{d}_\mu - \overleftarrow{d}_\mu R_\mu M\right) q_L\right.$$
$$+\bar{q}_L\left(L_\mu M^\dagger \overrightarrow{d}_\mu - \overleftarrow{d}_\mu L_\mu M^\dagger + M^\dagger R_\mu \overrightarrow{d}_\mu\right.$$
$$\left.\left. - \overleftarrow{d}_\mu M^\dagger R_\mu\right) q_R\right], \tag{47}$$

where the equality holds up to fully bosonized terms of order $1/\Lambda_\chi^4$. The last term can be easily translated into the correspondent *constituent* quark term; it corresponds to the sum of the terms 5. + 6. of the mixed sector in the list (50). Notice that this term represents interactions among scalar and vector fields. In addition, it is generally true that multifermion terms with more than four quarks require the introduction of more than a single field with given Lorentz properties (i.e. excited resonance states).
The possible relevance of additional non-renormalizable terms in the scalar sector of the NJL model has been already pointed out in [10]. They modify the mass-gap equation and can be incorporated in a renormalization of the scalar coupling G_S, or alternatively of the expectation value of the scalar field M_Q which minimizes the effective potential.

We proceed now to the classification of all the *constituent* quark-resonance bilinears which appear up to $\frac{1}{\Lambda_\chi^2}$ order (i.e. suppressed up to Λ_χ^2 power respect to the leading quark-resonance bilinears). They are all the quark-resonance bilinears which are locally chiral invariant, with the *caveats* already discussed. They can be generally represented by:

$$\left(\frac{1}{\Lambda_\chi}\right)^n \times R^k \times (\nabla, \nabla^C)^{n-k+1}, \tag{48}$$

with $n \leq 2$. k ranges from 0 to 3 and identifies four possible classes. R is a resonance from the set $\{W^+ \pm \gamma_5 W^-, \tilde{H}\}$ and ∇, ∇^C are the covariant derivatives defined in (33), (34).

We summarize in Table 1 the P and C transformation properties of the constituent quark bilinears and in Table 2 the P and C transformation properties of the fundamental fields in the R set together with the currents ξ_μ and Γ_μ.
We work in the chiral limit and we set to zero all terms that contain the fields Σ and Δ which are proportional to the quark mass matrix $\mathcal{M}$. The integration over quarks induces a mixing between the pseudoscalar field ξ_μ and the axial field W_μ^- which is leading in the chiral expansion. The physical fields are obtained after a diagonalization of the quadratic matrix. In the ENJL model this leads to a rescaling of the pseudoscalar field by the mixing parameter g_A, which the authors of [8] connect to the g_A parameter of the effective quark-model by Georgi-Manohar [16]. In the QR model the mixing parameter g_A is affected by higher order corrections: the physical pseudoscalar field is defined by the rescaling

$$\xi_\mu \to g'_A \xi_\mu, \tag{49}$$

with a new mixing parameter g'_A. In the following the field ξ_μ will be the physical field defined in (49).

Table 1. Parity and Charge Conjugation transformation properties of the quark bilinears

	P	C
$\bar{Q}Q$	+	+
$\bar{Q}\gamma_5 Q$	−	+
$\bar{Q}\gamma_\mu\gamma_5 Q$	$-\epsilon(\mu)$	$(\bar{Q}\gamma_\mu\gamma_5 Q)^T$
$\bar{Q}\gamma_\mu Q$	$\epsilon(\mu)$	$-(\bar{Q}\gamma_\mu Q)^T$
$\bar{Q}\sigma_{\mu\nu}Q$	$\epsilon(\mu)\epsilon(\nu)$	$-(\bar{Q}\sigma_{\mu\nu}Q)^T$
$\bar{Q}\sigma_{\mu\nu}\gamma_5 Q$	$\sim \epsilon^{\mu\nu\alpha\beta}V_{\alpha\beta}$	

Table 2. Parity and Charge Conjugation transformation properties of the fundamental fields of the effective meson theory

	P	C
V_μ	$\epsilon(\mu)$	$-V_\mu^T$
A_μ	$-\epsilon(\mu)$	A_μ^T
σ	σ	σ^T
Γ_μ	$\epsilon(\mu)$	$-\Gamma_\mu^T$
ξ_μ	$-\epsilon(\mu)$	ξ_μ^T
$f_{\mu\nu}^\pm$	$\pm\epsilon(\mu)\epsilon(\nu)$	$\mp f_{\mu\nu}^{\pm\,T}$
$\chi_\pm$	$\pm\chi_\pm$	$\chi_\pm^T$

At order $\frac{1}{\Lambda_\chi}$ there are not invariants coming from the bosonization of the most general non renormalizable Lagrangian with *current* quarks.

In terms of the linear combinations of the axial and vector fields $\hat{W}^\pm = W^+ \pm \gamma_5 W^-$ all possible invariants at $1/\Lambda_\chi^2$ order are:

1. $\bar{Q}\gamma_\mu[\overrightarrow{\nabla}^\lambda, [\overrightarrow{\nabla}_\mu, \overrightarrow{\nabla}_\lambda]]Q$
2. $\bar{Q}\gamma_\mu\{\overrightarrow{\nabla}_\mu, \overrightarrow{\nabla}^2\}Q$

1. $\bar{Q}\gamma_\mu\left\{[\overrightarrow{\nabla}_\nu, \hat{W}_\mu^-]\overrightarrow{\nabla}^\nu - \overleftarrow{\nabla}_\nu [\overrightarrow{\nabla}_\nu, \hat{W}_\mu^-]\right\}Q$
2. $\bar{Q}\gamma_\mu\{\hat{W}_\mu^- \overrightarrow{\nabla}^2\}Q$
3. $\bar{Q}\gamma_\mu\{\{\overrightarrow{\nabla}_\mu, \overrightarrow{\nabla}_\nu\}\hat{W}_\nu^-\}Q$
4. $\bar{Q}\gamma_\mu(\overleftarrow{\nabla}_\mu \hat{W}_\nu^- \overrightarrow{\nabla}_\nu + \overleftarrow{\nabla}_\nu \hat{W}_\nu^- \overrightarrow{\nabla}_\mu)Q$
5. $\bar{Q}\gamma_\mu[[\overrightarrow{\nabla}_\mu, \overrightarrow{\nabla}_\nu], \hat{W}_\nu^-]Q$
6. $\bar{Q}\gamma_\mu\{\overrightarrow{\nabla}_\mu, \hat{W}^{-2}\}Q$
7. $\bar{Q}\gamma_\mu([\overrightarrow{\nabla}_\mu, \hat{W}_\nu^-]\hat{W}_\nu^- + \hat{W}_\nu^-[\hat{W}_\nu^-, \overrightarrow{\nabla}_\mu])Q$
8. $\bar{Q}\gamma_\mu(\hat{W}_\mu^- \hat{W}_\nu^- \overrightarrow{\nabla}_\nu + \overleftarrow{\nabla}_\nu \hat{W}_\nu^- \hat{W}_\mu^-)Q$

9. $\bar{Q}\gamma_\mu([\vec{\nabla}_\nu, \hat{W}_\mu^- \hat{W}_\nu^-] - [\vec{\nabla}_\nu, \hat{W}_\nu^- \hat{W}_\mu^-])Q$

10. $\bar{Q}\gamma_\mu([\vec{\nabla}_\nu, \hat{W}_\mu^-]\hat{W}_\nu^- + \hat{W}_\nu^-[\hat{W}_\mu^-, \vec{\nabla}_\nu])Q$

11. $\bar{Q}\gamma_\mu\{\hat{W}^{-2}, \hat{W}_\mu^-\}Q$

1. $\bar{Q}_L \tilde{H}^3 Q_R + h.c.$
2. $\bar{Q}\gamma_\mu\{\vec{\nabla}_\mu, \tilde{H}^2\}Q$
3. $\bar{Q}\gamma_\mu \tilde{H}\, \vec{\nabla}_\mu\, \tilde{H} Q$
4. $\bar{Q}(\tilde{H}\, \vec{\nabla}^2 + \overleftarrow{\nabla}^{C^T 2}\, \tilde{H})Q$
5. $\bar{Q}\, \overleftarrow{\nabla}_\mu^{C^T}\, \tilde{H}\, \vec{\nabla}_\mu\, Q$

1. $\bar{Q}(\tilde{H}\hat{W}^{-2} + \hat{W}^{+2}\tilde{H})Q$
2. $\bar{Q}\hat{W}_\mu^+ \tilde{H} \hat{W}_\mu^- Q$
3. $\bar{Q}\gamma_\mu\{\hat{W}_\mu^-, \tilde{H}^2\}Q$
4. $\bar{Q}\gamma_\mu \tilde{H}\hat{W}_\mu^+ \tilde{H} Q$
5. $\bar{Q}(\hat{W}_\mu^+ \tilde{H}\, \vec{\nabla}_\mu - \overleftarrow{\nabla}_\mu^{C^T}\, \tilde{H}\hat{W}_\mu^-)Q$
6. $\bar{Q}(\tilde{H}\hat{W}_\mu^-\, \vec{\nabla}_\mu - \overleftarrow{\nabla}_\mu^{C^T}\, \hat{W}_\mu^+ \tilde{H})Q$
7. $$\bar{Q}[\hat{W}_\mu^+(\overleftarrow{\nabla}_\mu^{C^T}\, \tilde{H} + \tilde{H}\, \vec{\nabla}_\mu) - (\overleftarrow{\nabla}_\mu^{C^T}\, \tilde{H} + \tilde{H}\, \vec{\nabla}_\mu)\hat{W}_\mu^-]Q, \tag{50}$$

where we have used the hermiticity of the scalar field $\tilde{H} = \tilde{H}^\dagger$. We have grouped the terms into four classes according to the types of interactions among resonances.

The first class contains two independent terms which are totally derivative: the first is totally antisymmetric and is proportional to the field strenghts of Γ_μ and ξ_μ currents defined in (26) through the identity

$$[\vec{\nabla}_\mu, \vec{\nabla}_\lambda] = iG_{\mu\lambda} + \Gamma_{\mu\lambda} - \frac{i}{2}\gamma_5 \xi_{\mu\lambda} - \frac{1}{4}[\xi_\mu, \xi_\lambda], \tag{51}$$

where $G_{\mu\lambda} = \partial_\mu G_\lambda - \partial_\lambda G_\mu + i[G_\mu, G_\lambda]$. The second term acts as a renormalization of the fermion propagator $\partial_\mu \to \partial_\mu(1 + \partial^2/\Lambda_\chi^2)$. The second class is the Vector set and contains interactions among vector and axial vector fields $W_\mu^\pm$ with pseudoscalar mesons through the covariant derivatives ∇, ∇^C. The first three terms of this set enter the calculation of the two-point vector Green's function of Sect. (3.4). The third class is the Scalar set which contains interactions among scalars and interactions among scalars and pseudoscalar mesons. The last set is the mixed Vector-Scalar sector. We have neglected corrections of order M_Q^2/Λ_χ^2 where M_Q is the vev of the scalar field $\tilde{H}$.

In the following sections, after having classified the types of next-to-leading corrections which can be generated by the operators of the list (50), we will focus on the vector meson Lagrangian and more specifically on the numerical contributions of higher dimensional operators to the two-point vector Green's function.

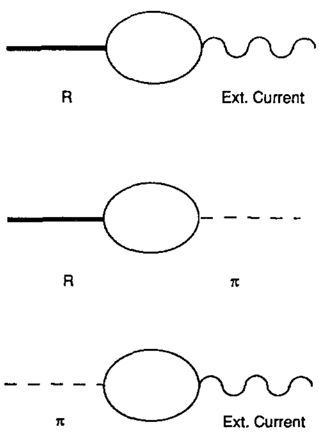

Fig. 2. A quark-loop diagram with at least one meson field as external leg. The integration over quarks (and gluons) produces the vertices of the effective meson Lagrangian. Double lines are resonances, dotted lines are pions and wavy lines are the external currents

3 The effective meson Lagrangian

The effective meson theory is given by the integral over quarks and gluons of the Lagrangian (37). By neglecting gluon corrections, which are inessential to our argument, the derivation of the low energy theory reduces to the integral over constituent quarks of the quark-resonance effective Lagrangian:

$$\int\int \mathscr{D}Q\mathscr{D}\bar{Q} \exp\left[\int d^4x \left(\bar{Q}\gamma^\mu(\partial_\mu + iG_\mu)Q + \sum_0^\infty \left(\frac{1}{\Lambda_\chi}\right)^n \bar{Q}RQ\right)\right] \equiv \det\left[\hat{D}_0 + \sum_0^\infty \left(\frac{1}{\Lambda_\chi}\right)^n R\right], \tag{52}$$

where $\hat{D}_0 = \gamma^\mu(\partial_\mu + iG_\mu)$ is the free fermion operator. The fermionic determinant generates the set of one quark-loop diagrams which mediate the interactions among the meson fields as shown in Fig. 2. Higher dimensional terms contain powers of $\partial^2/\Lambda_\chi^2$ i.e. of derivatives on internal quarks or external mesons.

The leading terms of the ENJL model have a logarithmic dependence upon the cutoff Λ_χ. Terms without logarithms can receive contributions from all higher order terms. Indeed, besides the finite contributions of the leading renormalizable operators, higher dimensional non-renormalizable operators differing from the leading ones by powers of derivatives may develop divergences that, integrated up to the cutoff Λ_χ, do compensate the inverse powers of Λ_χ and contribute as constant terms. The same happens to the terms which are of order $1/\Lambda_\chi^2$ in the final low energy meson Lagrangian: only those accompanied by logarithms can be traced back to terms of order $1/\Lambda_\chi^2$ in the original quark-resonance Lagrangian while those without logarithms are determined by the whole tower of non-renormalizable interactions. In logarithmic terms also the derivatives on internal quarks turn into powers of external momenta.

We will limit the rest of our discussion to the sector of the quark-resonance model which gives contribution to the parameters of the vector resonance Lagrangian already present at leading order.
The analysis shows that higher order contributions cannot be reabsorbed in a redefinition of the independent parameters

of the leading order. This implies that relations among resonance parameters valid at zero energy (i.e. at the leading order) can be modified when the energy increases (i.e. including next-to-leading corrections). Nevertheless the *caveats* on the equivalence between the current and constituent quark Lagrangians highly constrain the next-to-leading corrections to low energy QCD relations among vector, axial, scalar and pseudoscalar Green's functions which are valid in the leading ENJL model.

As already discussed, we will collect only next-to-leading power to leading log corrections (NPLL) of order $\frac{Q^2}{\Lambda_\chi^2} \ln \frac{\Lambda_\chi^2}{Q^2}$, which receive contribution from a *finite set* of higher dimensional operators (only $\frac{1}{\Lambda_\chi^2}$ terms).

The coefficients $\beta(\Lambda_\chi)$ of the new $1/\Lambda_\chi^2$ terms have to be fixed from experimental data.

3.1 The vector meson Lagrangian

The leading non anomalous Lagrangian with one vector meson (i.e. of order p^3) is:

$$\begin{aligned}\mathscr{L}_V = &-\frac{1}{4} < V_{\mu\nu}V^{\mu\nu} > +\frac{1}{2}M_V^2 < V_\mu V^\mu > \\ &-\frac{f_V}{2\sqrt{2}} < V_{\mu\nu}f_+^{\mu\nu} > -i\frac{g_V}{2\sqrt{2}} < V_{\mu\nu}[\xi^\mu,\xi^\nu] > \\ &+H_V < V_\mu[\xi_\nu, f_-^{\mu\nu}] > +iI_V < V_\mu[\xi^\mu,\chi_-] > \end{aligned} \quad (53)$$

and corresponds to the so called conventional vector model [6, 7], where the vector fields are introduced as ordinary fields. This is the natural form for the effective low energy theory after the bosonization of four-fermion interactions. In the chiral limit the I_V term is zero and the Lagrangian is parametrized by five constants: the vector resonance wave function renormalization constant Z_V, the mass M_V and the coupling constants f_V, g_V and H_V.

The ENJL estimate of the five parameters has been already derived in [8, 17] by using the heat kernel expansion technique for the calculation of the fermion determinant. Both the leading and non-leading contributions can be rederived by using the loopwise expansion. The fermion differential operator is a sum of the free part D_0 and a perturbation δ, which contains the long-wavelenght boson fields and powers of derivatives and the euclidean effective action can be written as:

$$\begin{aligned}\Gamma_{\text{eff}}(\delta) &= -\text{Tr}\ln[D_0+\delta] + \text{Tr}\ln D_0 \\ &= -\text{Tr}D_0^{-1}\delta + \frac{1}{2}\text{Tr}(D_0^{-1}\delta)^2 - \frac{1}{3}\text{Tr}(D_0^{-1}\delta)^3 + ...,\end{aligned} \quad (54)$$

where we have subtracted its value at $\delta = 0$.

The various terms on the rhs are identified by the order n in the series expansion of the logarithm. The term $\text{Tr}D_0^{-1}\delta$ (n=1) contains the tadpole graphs. The next term (n=2) contains the set of graphs with the insertion of two vertices in the loop and so on. The contributions to the parameters of $\mathscr{L}_V$ of (53) arise from the $n = 2$ and $n = 3$ insertions of vertices in the perturbative expansion.
At leading order and in the chiral limit δ is given by:

$$\delta = \delta_0 = \gamma_\mu[\Gamma_\mu - \frac{i}{2}W_\mu^+ - \frac{i}{2}\gamma_5(\xi_\mu - W_\mu^-)], \quad (55)$$

and the free part D_0 is

$$D_0 = \gamma_\mu(\partial_\mu + iG_\mu - M_Q). \quad (56)$$

The mass term M_Q acts as an infrared cutoff in the quark loop diagrams.
The complete operator δ is the sum of the leading part δ_0 defined in (55) and the non leading contributions in the $1/\Lambda_\chi$ expansion:

$$\delta = \delta_0 + \sum_{n=1}^{\infty}\left(\frac{1}{\Lambda_\chi}\right)^n R. \quad (57)$$

In Appendix A the one quark-loop diagrams with n=2 are explicitely calculated with the insertion of a generic form of the operator $\delta(x)$. Using those formulas one can get the contribution to a given parameter of the vector Lagrangian with the substitution of the appropriate operator $\delta(x)$. The next order (n=3) is calculated in Appendix B for the case which enter the calculation of the parameters that are analyzed in detail in Sects. 3.4 and 4.

3.2 The Leading contributions

The leading contributions to the parameters Z_V, M_V, f_V, g_V, I_V and H_V are obtained by the δ_0 insertion in the loopwise expansion. Z_V and M_V terms have the form $2R \times \nabla, \nabla^C$, while g_V, f_V and H_V have the form $1R \times \nabla, \nabla^C$, with the use of identities (28), (51). The mass term I_V is of the type $1R \times \nabla, \nabla^C \times \Delta$, with Δ defined in (29).

Z_V (or equivalently M_V) receives contribution from the n=2 diagram with the insertion of two vector fields:

$$-\frac{i}{2}\gamma_\mu W_\mu^+ \times -\frac{i}{2}\gamma_\mu W_\mu^+, \quad (58)$$

while $n = 3$ and $n = 4$ diagrams with the addition of the $\gamma_\mu\Gamma_\mu$ vertex add to the previous term to form a covariant expression. g_V and f_V keep contribution from the n=2 diagram:

$$-\frac{i}{2}\gamma_\mu W_\mu^+ \times \gamma_\mu\Gamma_\mu. \quad (59)$$

Contributions to g_V and H_V come also from the n=3 diagram:

$$-\frac{i}{2}\gamma_\mu W_\mu^+ \times -\frac{i}{2}\gamma_\mu\gamma_5\xi_\mu \times -\frac{i}{2}\gamma_\mu\gamma_5\xi_\mu. \quad (60)$$

Finally the I_V term comes from the n=3 diagram:

$$-\frac{i}{2}\gamma_\mu W_\mu^+ \times -\frac{i}{2}\gamma_\mu\gamma_5\xi_\mu \times \frac{1}{2}\gamma_5\Delta. \quad (61)$$

The leading divergent contributions to the five parameters of the vector Lagrangian are given by:

$$
\begin{aligned}
Z_V &= \frac{N_c}{16\pi^2} 2 \int_0^1 d\alpha\alpha(1-\alpha) \ln \frac{\Lambda^2}{S(\alpha)} \\
M_V^2 &= \frac{N_c}{16\pi^2}\left(\frac{\Lambda_\chi^2}{2G_V}\right)\frac{1}{Z_V} \\
f_V &= \sqrt{2}\sqrt{Z_V} \\
g_V &= \frac{N_c}{16\pi^2}\sqrt{2}(1-g_A^2)\frac{1}{\sqrt{Z_V}} \int_0^1 d\alpha\alpha(1-\alpha) \ln \frac{\Lambda^2}{S(\alpha)} \\
H_V &= -i\frac{N_c}{16\pi^2} g_A^2 \frac{1}{\sqrt{Z_V}} \int_0^1 d\alpha\alpha(1-\alpha) \ln \frac{\Lambda^2}{S(\alpha)}. \qquad (62)
\end{aligned}
$$

The function $S(\alpha)$ is equal to $M_Q^2+\alpha(1-\alpha)Q^2$ and depends explicitely upon the external momentum Q^2. At $Q^2 = 0$, one recovers the low energy limit of the ENJL model derived in [8], where the values of the parameters are the following:

$$
\begin{aligned}
Z_V &= \frac{N_c}{16\pi^2}\frac{1}{3}\ln\frac{\Lambda^2}{M_Q^2} \\
M_V^2 &= \frac{N_c}{16\pi^2}\left(\frac{\Lambda_\chi^2}{2G_V}\right)\frac{1}{Z_V} \\
f_V &= \sqrt{2Z_V} \\
g_V &= \frac{N_c}{16\pi^2}\frac{\sqrt{2}}{6}(1-g_A^2)\frac{1}{\sqrt{Z_V}}\ln\frac{\Lambda^2}{M_Q^2} \\
H_V &= -i\frac{N_c}{16\pi^2}\frac{g_A^2}{6}\frac{1}{\sqrt{Z_V}}\ln\frac{\Lambda^2}{M_Q^2}. \qquad (63)
\end{aligned}
$$

They coincide with the ones calculated in [8] in the proper time regularization scheme, where one has to use the expression of the incomplete Gamma function $\Gamma(0, x = \frac{M_Q^2}{\Lambda_\chi^2}) = -\ln x - \gamma_E + \mathcal{O}(x)$ for small values of x.

The leading contributions to the parameters of the vector meson Lagrangian are all logarithmically divergent. Furthermore the five parameters are not all independent. They can be expressed in terms of three of the input parameters of the ENJL model:

$$
x = \frac{M_Q^2}{\Lambda_\chi^2},\ G_V,\ g_A. \qquad (64)
$$

As we will see in the next section this reduction of the number of independent parameters does not hold at next-to-leading order.

3.3 The Next-to-Leading contributions

As already discussed, we will restrict to the NPLL corrections $\frac{Q^2}{\Lambda_\chi^2}\ln\frac{\Lambda_\chi^2}{Q^2}$ generated by the insertion of higher dimensional $1/\Lambda_\chi^2$ vertices.

In order to determine how many independent parameters we are left with after the inclusion of non-renormalizable interactions (NRI) in the quark-resonance Lagrangian, we analyze the corresponding vertices that give contribution to the five parameters of the Lagrangian $\mathscr{L}_V$ at next-to-leading order. There are seven $1/\Lambda_\chi^2$ terms which can contribute to the five vector resonance parameters. They are the two totally derivative terms and the five one vector terms in the list (50). Their contributions come from n=2 and n=3 cases of the loopwise expansion and $n > 3$ terms reconstruct the covariant form.

For $n = 2$ the sets of pairs (a, b) of vertices $\{V^a\}\times\{V^b\}$, contributing to each parameter of (53) are the following:

$$
\begin{aligned}
&Z_V(M_V) \Leftrightarrow \left\{\frac{i}{2}\gamma_\mu W^{+\mu}\right\} \times \left\{\left[\frac{\beta_V^1}{\Lambda_\chi^2}\gamma_\mu\right.\right. \\
&\left([\vec{\nabla}_\nu, \hat{W}_\mu^-]\,\vec{\nabla}^\nu - \vec{\nabla}_\nu\,[\vec{\nabla}_\nu, \hat{W}_\mu^-]\right) \\
&\left.\left.+\frac{\beta_V^2}{\Lambda_\chi^2}\gamma_\mu\{\hat{W}_\mu^-, \vec{\nabla}^2\} + \frac{\beta_V^3}{\Lambda_\chi^2}\gamma_\mu\{\{\vec{\nabla}_\mu, \vec{\nabla}_\nu\}, \hat{W}_\nu^-\}\right]\right\} \\
&f_V \Leftrightarrow \left\{\gamma_\mu\Gamma^\mu\right\} \times \left\{\left[\frac{\beta_V^1}{\Lambda_\chi^2}\gamma_\mu\right.\right. \\
&\left([\vec{\nabla}_\nu, \hat{W}_\mu^-]\,\vec{\nabla}^\nu - \vec{\nabla}_\nu\,[\vec{\nabla}_\nu, \hat{W}_\mu^-]\right) \\
&\left.\left.+\frac{\beta_V^2}{\Lambda_\chi^2}\gamma_\mu\{\hat{W}_\mu^-, \vec{\nabla}^2\} + \frac{\beta_V^3}{\Lambda_\chi^2}\gamma_\mu\{\{\vec{\nabla}_\mu, \vec{\nabla}_\nu\}, \hat{W}_\nu^-\}\right]\right\} + \\
&\left\{\frac{i}{2}\gamma_\mu W^{+\mu}\right\} \times \left\{\frac{\beta_\Gamma^1}{\Lambda_\chi^2}\gamma_\mu[\vec{\nabla}^\lambda, [\vec{\nabla}_\mu, \vec{\nabla}_\lambda]]\right\} \\
&g_V \Leftrightarrow \{\text{those of} f_V\} + \left\{\frac{i}{2}\gamma_\mu\gamma_5\xi^\mu\right\} \\
&\times\left\{\left[\frac{\beta_V^1}{\Lambda_\chi^2}\gamma_\mu\left([\vec{\nabla}_\nu, \hat{W}_\mu^-]\,\vec{\nabla}^\nu - \vec{\nabla}_\nu\,[\vec{\nabla}_\nu, \hat{W}_\mu^-]\right)\right.\right. \\
&+\frac{\beta_V^2}{\Lambda_\chi^2}\gamma_\mu\{\hat{W}_\mu^-, \vec{\nabla}^2\} + \frac{\beta_V^3}{\Lambda_\chi^2}\gamma_\mu\{\{\vec{\nabla}_\mu, \vec{\nabla}_\nu\}, \hat{W}_\nu^-\} \\
&\left.\left.+\frac{\beta_V^4}{\Lambda_\chi^2}\gamma_\mu(\vec{\nabla}_\mu\,\hat{W}_\nu^-\,\vec{\nabla}_\nu + \vec{\nabla}_\nu\,\hat{W}_\nu^-\,\vec{\nabla}_\mu)\right]\right\} \\
&H_V \Leftrightarrow \left\{\frac{i}{2}\gamma_\mu W^{+\mu}\right\} \times \left\{\frac{\beta_\Gamma^1}{\Lambda_\chi^2}\gamma_\mu[\vec{\nabla}^\lambda, [\vec{\nabla}_\mu, \vec{\nabla}_\lambda]]\right\} + \\
&\left\{\frac{i}{2}\gamma_\mu\gamma_5\xi^\mu\right\} \times \left\{\frac{\beta_V^5}{\Lambda_\chi^2}\gamma_\mu[[\vec{\nabla}_\mu, \vec{\nabla}_\nu], \hat{W}_\nu^-]\right\}. \qquad (65)
\end{aligned}
$$

In order to reduce the number of independent terms, we have used the transversality condition on the massive vecor field $d_\mu W^\mu = 0$.

The contributions at $n = 3$ in a notation $\{V^a\}\times\{V^b\}\times\{V^c\}$ are:

$$
\begin{aligned}
&Z_V(M_V) \Leftrightarrow \left\{\frac{i}{2}\gamma_\mu W^{+\mu}\right\} \times \left\{\frac{i}{2}\gamma_\nu W^{+\nu}\right\} \\
&\times\left\{\frac{\beta_\Gamma^2}{\Lambda_\chi^2}\gamma_\lambda\{\vec{\nabla}_\lambda, \vec{\nabla}^2\}\right\} \\
&f_V \Leftrightarrow \left\{\frac{i}{2}\gamma_\mu W^{+\mu}\right\} \times \left\{\gamma_\nu\Gamma^\nu\right\} \times \left\{\frac{\beta_\Gamma^2}{\Lambda_\chi^2}\gamma_\lambda\{\vec{\nabla}_\lambda, \vec{\nabla}^2\}\right\} \\
&g_V \Leftrightarrow \{\text{those of } f_V\} + \left\{\frac{i}{2}\gamma_\mu W^{+\mu}\right\} \\
&\times\left\{\frac{i}{2}\gamma_\nu\gamma_5\xi^\nu\right\} \times \left\{\frac{\beta_\Gamma^2}{\Lambda_\chi^2}\gamma_\lambda\{\vec{\nabla}_\lambda, \vec{\nabla}^2\}\right\} + \\
&\left\{\frac{i}{2}\gamma_\mu\gamma_5\xi^\mu\right\} \times \left\{\frac{i}{2}\gamma_\nu\gamma_5\xi^\nu\right\}
\end{aligned}
$$

$$\times\left\{\left[\frac{\beta_V^1}{\Lambda_\chi^2}\gamma_\mu\left([\vec\nabla_\nu,\hat W_\mu^-]\,\vec\nabla^\nu-\vec\nabla_\nu\,[\vec\nabla_\nu,\hat W_\mu^-]\right)\right.\right.$$
$$\left.\left.+\frac{\beta_V^2}{\Lambda_\chi^2}\gamma_\mu\{\hat W_\mu^-,\vec\nabla^2\}+\frac{\beta_V^3}{\Lambda_\chi^2}\gamma_\mu\{\{\vec\nabla_\mu,\vec\nabla_\nu\},\hat W_\nu^-\}\right]\right\}$$
$$H_V\Leftrightarrow\left\{\frac{i}{2}\gamma_\mu W^{+\mu}\right\}\times\left\{\frac{i}{2}\gamma_\nu\gamma_5\xi^\nu\right\}\times\left\{\frac{\beta_\Gamma^2}{\Lambda_\chi^2}\gamma_\lambda\{\vec\nabla_\lambda,\vec\nabla^2\}\right\}$$
$$+\left\{\frac{i}{2}\gamma_\mu\gamma_5\xi^\mu\right\}\times\left\{\frac{i}{2}\gamma_\nu\gamma_5\xi^\nu\right\}$$
$$\times\left\{\left[\frac{\beta_V^1}{\Lambda_\chi^2}\gamma_\mu\left([\vec\nabla_\nu,\hat W_\mu^-]\,\vec\nabla^\nu-\vec\nabla_\nu\,[\vec\nabla_\nu,\hat W_\mu^-]\right)\right.\right.$$
$$\left.\left.+\frac{\beta_V^2}{\Lambda_\chi^2}\gamma_\mu\{\hat W_\mu^-,\vec\nabla^2\}+\frac{\beta_V^3}{\Lambda_\chi^2}\gamma_\mu\{\{\vec\nabla_\mu,\vec\nabla_\nu\},\hat W_\nu^-\}\right]\right\}. \tag{66}$$

Each diagram has one (or two) leading vertex and one NTL vertex. At next-to-leading order in the $1/\Lambda_\chi$ expansion the five chiral leading vector resonance parameters depend upon 10 free coefficients at most. Three come from the leading order and seven from $1/\Lambda_\chi^2$ terms. Some of the contributions are zero, as we will see in the next sections. Inspite of this reduction the five vector parameters become all independent at NTL order and acquires a dependence upon Q^2 of the form:

$$f_i=\left(1+\beta_i\frac{Q^2}{\Lambda_\chi^2}\right)\ln\frac{\Lambda_\chi^2}{Q^2}. \tag{67}$$

3.4 The running of f_V^2 and M_V^2

For a detailed evaluation of the NTL contributions we concentrate on two of the five parameters of the vector Lagrangian relevant for the behaviour of the two-point vector Green function that we will compare with experimental data in Sect. 4: the coupling f_V between the vector meson and the external vector current and the mass M_V. In the ENJL model the two parameters are both expressed in terms of the wave function renormalization constant Z_V as follows:

$$f_V=\sqrt{2Z_V}\quad M_V^2=\frac{N_c}{16\pi^2}\left(\frac{\Lambda_\chi^2}{2G_V}\right)\frac{1}{Z_V}, \tag{68}$$

where Z_V is the leading logarithmic contribution to the wave-function

$$Z_V=Z_V^l=2\frac{N_c}{16\pi^2}\int_0^1 d\alpha\alpha(1-\alpha)\ln\frac{\Lambda_\chi^2}{s(\alpha)}. \tag{69}$$

The product $f_V^2M_V^2$ is scale invariant:

$$f_V^2M_V^2=\frac{N_c}{16\pi^2}\frac{\Lambda_\chi^2}{G_V}\,. \tag{70}$$

By adding the NPLL corrections, the f_V coupling receives contributions which are absent in the wave function Z_V. The latter defines the renormalized vector mass M_V, once the physical vector field has been introduced.

The full Lagrangian up to $1/\Lambda_\chi^2$ order which gives contribution to f_V and Z_V (or equivalently to M_V) is:

$$\mathscr{L}=\bar Q(\hat\partial-M_Q)Q+\bar Q\gamma_\mu\Gamma_\mu Q-\frac{i}{2}\bar Q\gamma_\mu W_\mu^+Q$$
$$+\frac{\beta_\Gamma^1}{\Lambda_\chi^2}\bar Q\gamma_\mu d^\lambda\Gamma_{\mu\lambda}Q+\frac{\beta_\Gamma^2}{\Lambda_\chi^2}\bar Q\gamma_\mu\{\vec d_\mu,\vec d^{\,2}\}Q$$
$$+\frac{i}{2}\frac{\beta_V^1}{\Lambda_\chi^2}\bar Q\gamma_\mu d^2W_\mu^+Q+\frac{i}{2}\frac{\beta_V^2}{\Lambda_\chi^2}\bar Q\gamma_\mu\{d^2,W_\mu^+\}Q$$
$$+\frac{i}{2}\frac{\beta_V^3}{\Lambda_\chi^2}\bar Q\gamma_\mu\{W_\nu^+,\{d_\mu,d_\nu\}\}Q. \tag{71}$$

The first term defines the inverse free fermion propagator $D_0=\hat\partial-M_Q$. The rest defines the local perturbation $\delta(x)$ up to $1/\Lambda_\chi^2$. There are five $1/\Lambda_\chi^2$ terms with new coefficients β_i. Each term can be traced back to the corresponding term in the list (66) where the covariant derivative d_μ is defined in terms of the covariant derivative ∇_μ as follows:

$$\nabla_\mu=\partial_\mu+\Gamma_\mu-\frac{i}{2}\gamma_5\xi_\mu\equiv d_\mu-\frac{i}{2}\gamma_5\xi_\mu. \tag{72}$$

The covariant derivative on the vector-like fields W_μ^+,Γ_μ is defined as:

$$d_\mu W_\nu^+=\partial_\mu W_\nu^++[\Gamma_\mu,W_\nu^+]. \tag{73}$$

The general formula resulting for f_V and M_V^2 can be written as follows:

$$f_V=\sqrt{2Z_V}+\frac{N_c}{16\pi^2}\frac{\sqrt2}{3}\frac{1}{\sqrt{Z_V}}\frac{Q^2}{\Lambda_\chi^2}\left[\sum_{i=1}^2\frac{\beta_\Gamma^i}{2}\int_0^1\right.$$
$$d\alpha P_i^\Gamma(\alpha)\ln\frac{\Lambda_\chi^2}{S(\alpha)}$$
$$\left.-\frac12\sum_{i=1}^3\beta_V^i\int_0^1 d\alpha P_i^V(\alpha)\ln\frac{\Lambda_\chi^2}{S(\alpha)}\right]$$
$$M_V^2=\frac{N_c}{16\pi^2}\left(\frac{\Lambda_\chi^2}{2G_V}\right)\frac{1}{Z_V}, \tag{74}$$

where the wave function renormalization constant Z_V is given by:

$$Z_V=\frac{N_c}{16\pi^2}\frac13\left[6\int_0^1 d\alpha\alpha(1-\alpha)\ln\frac{\Lambda_\chi^2}{S(\alpha)}\right.$$
$$+\sum_{i=1}^3\beta_V^i\frac{Q^2}{\Lambda_\chi^2}\int_0^1 d\alpha P_i^V(\alpha)\ln\frac{\Lambda_\chi^2}{S(\alpha)}$$
$$\left.+\frac32\beta_\Gamma^2\frac{Q^2}{\Lambda_\chi^2}\int_0^1 d\alpha P_2^\Gamma(\alpha)\ln\frac{\Lambda_\chi^2}{S(\alpha)}\right]$$
$$\equiv Z_V^l+\frac{N_c}{16\pi^2}\frac13\left[\sum_{i=1}^3\beta_V^i\frac{Q^2}{\Lambda_\chi^2}\int_0^1 d\alpha P_i^V(\alpha)\ln\frac{\Lambda_\chi^2}{S(\alpha)}\right.$$
$$\left.+\frac32\beta_\Gamma^2\frac{Q^2}{\Lambda_\chi^2}\int_0^1 d\alpha P_2^\Gamma(\alpha)\ln\frac{\Lambda_\chi^2}{S(\alpha)}\right]. \tag{75}$$

The $\beta_{V,\Gamma}^i$ coefficients must be determined from experimental data. The contribution from β_Γ^2 enters at n=3, while the others enter at n=2. The function $S(\alpha)$ is equal to $M_Q^2+\alpha(1-\alpha)Q^2$. The $P_i^{V,\Gamma}(\alpha)$ are polynomials in the Feynman parameter α. Their explicit form can be derived

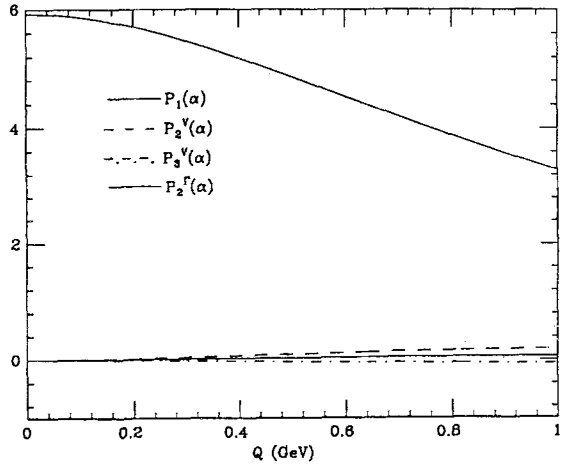

Fig. 3. The integrals $\int_0^1 d\alpha P_i(\alpha)\ln(\Lambda_\chi^2/s(\alpha))$ which occur in the NTL logarithmic corrections to the effective meson Lagrangian are shown as a function of $\sqrt{Q^2}$. The three polynomials correspond to the three cases of Appendix A

by the formulas in Appendix A (terms $\beta_V^i, \beta_\Gamma^1$) and the formula in Appendix B (term β_Γ^2). They read:

$$
\begin{aligned}
P_1^V(\alpha) &= P_1^\Gamma(\alpha) = 12\alpha(1-\alpha)\\
P_2^V(\alpha) &= \frac{3}{2}[8\alpha(1-\alpha) - 16\alpha^2(1-\alpha) - 36\alpha^2(1-\alpha)^2\\
&\quad +24\alpha^3(1-\alpha)]\\
P_3^V(\alpha) &= 6[3\alpha^2(1-\alpha)^2 - 2\alpha^3(1-\alpha)]\\
P_2^\Gamma(\alpha) &= -\frac{2}{3}[36\alpha^3(1-\alpha)^2 - 18\alpha^4(1-\alpha)]. \qquad (76)
\end{aligned}
$$

The dependence upon Q^2 of the quantity $\int_0^1 d\alpha P_i(\alpha)$ $\ln(\Lambda_\chi^2/S(\alpha))$ for the different P_i is shown in Fig. 3. From (76) one obtains that the purely divergent contribution (i.e. $\ln \Lambda_\chi^2/M_Q^2 \int_0^1 d\alpha P_i(\alpha)$) of $\beta_V^2, \beta_V^3, \beta_\Gamma^2$ terms is identically zero. Higher order power corrections produce a residual Q^2 dependence for the integrals of P_2^V, P_3^V, P_2^Γ as shown in Fig. 3.

We are left with two new coefficients $\beta_\Gamma^1, \beta_V^1$. The product (70) is now given by:

$$
\begin{aligned}
f_V^2 M_V^2 &= \frac{N_c}{16\pi^2}\frac{\Lambda_\chi^2}{G_V}\left[1 + \frac{N_c}{16\pi^2}\frac{1}{3}\frac{1}{Z_V}\frac{Q^2}{\Lambda_\chi^2}\left(2\frac{\beta_\Gamma^1}{2}\int_0^1\right.\right.\\
&\left.\left. d\alpha P_1(\alpha)\ln\frac{\Lambda_\chi^2}{S(\alpha)} - \beta_V^1\int_0^1 d\alpha P_1(\alpha)\ln\frac{\Lambda_\chi^2}{S(\alpha)}\right)\right], \qquad (77)
\end{aligned}
$$

where we have omitted the index V, Γ in $P_1(\alpha)$. The presence of the new NTL terms with coefficients β_Γ^1 and β_V^1 breaks in general the scale invariance of the product in (70).

4 Phenomenology of the vector-vector correlation function

To estimate the values of the $1/\Lambda_\chi^2$ coefficients which enter in the running of f_V and M_V^2 we focus on the particular channel of the vector resonance sector, by studying the Q^2 behaviour of the vector-vector correlation function where we can compare our predictions with the experimental results. We closely follow the derivation of the 2-point vector function of [12].

We define the 2-point vector function as:

$$
\Pi_{\mu\nu}^{V(ab)}(q^2) = i\int d^4x\, e^{iqx} < 0|T(V_\mu^a(x)V_\nu^b(0)|0 >, \qquad (78)
$$

where $V_\mu^a(x)$ is the flavoured vector quark current defined as:

$$
V_\mu^a(x) = \bar{q}(x)\gamma_\mu\frac{\lambda^a}{\sqrt{2}}q(x), \qquad (79)
$$

with λ^a the Gell-Mann matrices normalised as $tr(\lambda^a\lambda^b) = 2\delta^{ab}$. The Lorentz covariance and $SU(3)$ invariance imply for the $\Pi_{\mu\nu}^V$ the following structure:

$$
\begin{aligned}
\Pi_{\mu\nu}^{V(ab)}(q^2) &= (q_\mu q_\nu - g_{\mu\nu}q^2)\Pi_V^1(Q^2)\delta^{ab}\\
&\quad +q_\mu q_\nu \Pi_V^0(Q^2)\delta^{ab}, \qquad (80)
\end{aligned}
$$

where $Q^2 = -q^2$, with q^2 euclidean. The $SU(3)_L \times SU(3)_R$ ENJL model gives the low energy prediction for the invariant functions Π_V^1, Π_V^0 in the chiral limit ($\mathcal{M} \to 0$) and without the inclusion of chiral loops [12]:

$$
\begin{aligned}
\Pi_V^1(Q^2) &= -4(2H_1 + L_{10}) + \mathcal{O}(Q^2)\\
\Pi_V^0(Q^2) &= 0. \qquad (81)
\end{aligned}
$$

$\Pi_V^0(Q^2)$ is zero at all orders in the chiral limit.

The parameters H_1 and L_{10} are two of the twelve counterterms that appear in the non anomalous effective Lagrangian of pseudoscalar mesons at order p^4 in the chiral expansion:

$$
\begin{aligned}
\mathcal{L}_4 &= ...... + L_{10}tr(U^\dagger F_{\mu\nu}^R U F_L^{\mu\nu}) + H_1 tr(F_{\mu\nu R}^2 + F_{\mu\nu L}^2)\\
&= L_{10}\frac{1}{4}(f_{\mu\nu}^{+2} - f_{\mu\nu}^{-2}) + H_1\frac{1}{2}(f_{\mu\nu}^{+2} + f_{\mu\nu}^{-2}), \qquad (82)
\end{aligned}
$$

where $f_{\mu\nu}^\pm$ are related to the external field-strenght tensors $F_{\mu\nu}^{R,L}$ through the identity:

$$
f_{\mu\nu}^\pm = \xi F_{\mu\nu}^L \xi^\dagger \pm \xi^\dagger F_{\mu\nu}^R \xi \qquad (83)
$$

and

$$
\begin{aligned}
F_{\mu\nu}^L &= \partial_\mu l_\nu - \partial_\nu l_\mu - i[l_\mu, l_\nu]\\
F_{\mu\nu}^R &= \partial_\mu r_\nu - \partial_\nu r_\mu - i[r_\mu, r_\nu]. \qquad (84)
\end{aligned}
$$

The leading values of H_1 and L_{10} at $Q^2 = 0$ predicted by the QR model are:

$$
\begin{aligned}
H_1 &= -\frac{1}{12}\frac{N_c}{16\pi^2}(1 + g_A^2)\ln\frac{\Lambda_\chi^2}{M_Q^2} + \text{finite terms}\\
L_{10} &= -\frac{1}{6}\frac{N_c}{16\pi^2}(1 - g_A^2)\ln\frac{\Lambda_\chi^2}{M_Q^2} + \text{finite terms}. \qquad (85)
\end{aligned}
$$

The combination $2H_1 + L_{10}$ is free from finite contributions.

The vector-vector correlation function allows to explore a sector of the QR model which is free from the effects of the axial-pseudoscalar mixing (i.e. the parameter g_A). Indeed, the g_A^2 dependence is introduced by the $f_{\mu\nu}^-$ part of the invariant terms, which in turn depends on the ξ_μ physical

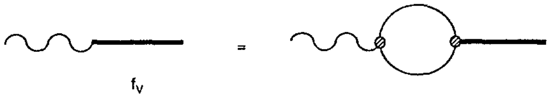

Fig. 4. The running of f_V with Q^2 generated by the QR model: the full circle indicates the insertion of a leading ($\mathcal{O}(1)$) or a next-to-leading ($\mathcal{O}(1/\Lambda_\chi^2)$) vertex in the one quark-loop diagram

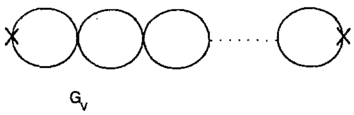

Fig. 5. The resummation of n-quark bubble diagrams which gives the full Q^2 dependence of the vector-vector correlation function in the ENJL model of [12]. They contain the insertion of the leading 4-quark vector vertex with coupling G_V

field because of the identity $f^-_{\mu\nu} = \xi_{\mu\nu}$. The vector two-point function gets contribution only from the $f^+_{\mu\nu}$ terms and therefore the parameters H_1 and L_{10} will only enter in a combination independent of g_A. The combination that appears in front of the $f^{+2}_{\mu\nu}$ term in the Lagrangian (82) is the following:

$$\frac{1}{4}(2H_1 + L_{10}) = -\frac{N_c}{16\pi^2}\frac{1}{12}\ln\frac{\Lambda_\chi^2}{M_Q^2} \tag{86}$$

and contributes as in (81) to the two-point vector correlation function. As was pointed out in [12], the vector resonance exchange also contributes to the Q^2 dependence of the $\Pi^1_V(Q^2)$ function. The total result is:

$$\Pi^1_V(Q^2) = -4(2H_1 + L_{10}) - 2\frac{f_V^2 Q^2}{M_V^2 + Q^2}, \tag{87}$$

which includes the contribution at $Q^2 = 0$ from the genuine one quark-loop diagram (first term) and the contribution from the vector resonance exchange (second term). In this approximation the parameters f_V and M_V are the values at $Q^2 = 0$ predicted by the ENJL model, i.e. they are generated by the single quark-loop diagrams with the insertion of leading vertices in the $1/\Lambda_\chi$-expansion (see Fig. 4).

In the ENJL model [8] at $Q^2 = 0$ the following relation holds:

$$(2H_1 + L_{10})(Q^2 = 0) = -\frac{f_V^2}{2}(Q^2 = 0) \tag{88}$$

so that the $\Pi^1_V(Q^2)$ function predicted by the ENJL model can be rewritten in a VMD way

$$\Pi^1_V(Q^2) = 2\frac{f_V^2 M_V^2}{M_V^2 + Q^2}, \tag{89}$$

where the parameters

$$f_V^2 = \frac{N_c}{16\pi^2}\frac{2}{3}\ln\frac{\Lambda_\chi^2}{M_Q^2}, \quad M_V^2 = \frac{3}{2}\frac{\Lambda_\chi^2}{G_V}\frac{1}{\ln\frac{\Lambda_\chi^2}{M_Q^2}} \tag{90}$$

are the values at $Q^2 = 0$ predicted by the ENJL model.

The authors of [12] have resummed all quark-bubble diagrams in Fig. 5 with the insertion of the leading 4-quark

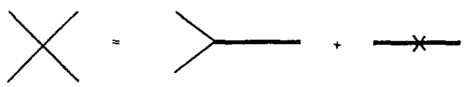

Fig. 6. The 4-quark vector vertex of the fermion action with coupling G_V is replaced by the sum of the q-q-vector vertex and the mass term of the vector field in the bosonized action

effective vertex with coupling G_V. In the VMD representation of (89), the Q^2 dependent contributions coming from the n-loop diagrams can be reabsorbed in the running of the vector parameters $f_V(Q^2)$ and $M_V^2(Q^2)$, which are completely determined in terms of the ENJL parameters. The result quoted in [12] is the following:

$$\Pi^1_V(Q^2) = 2\frac{f_V^2(Q^2)M_V^2(Q^2)}{M_V^2(Q^2) + Q^2}, \tag{91}$$

with

$$\begin{aligned} f_V^2(Q^2) &= \frac{N_c}{16\pi^2}4\int_0^1 d\alpha\alpha(1-\alpha) \\ &\quad \ln\frac{\Lambda_\chi^2}{M_Q^2 + \alpha(1-\alpha)Q^2} \\ M_V^2(Q^2) &= \frac{\Lambda_\chi^2}{4G_V}\frac{1}{\int_0^1 d\alpha\alpha(1-\alpha)\ln\frac{\Lambda_\chi^2}{M_Q^2+\alpha(1-\alpha)Q^2}}. \end{aligned} \tag{92}$$

In the formula (92) we kept only the leading logarithmic contribution of the expansion of the incomplete Gamma function $\Gamma(0,x) \simeq -\ln x - \gamma_E + \mathcal{O}(x)$ appearing in the calculation of [12].

In this case the product $f_V^2(Q^2)M_V^2(Q^2)$ remains scale invariant.

4.1 $\Pi^1_V(Q^2)$ *from the QR model*

The full Q^2 dependence of the vector-vector function can be extracted from the bosonized generating functional. In this case pure fermion vertices are absent and in particular the 4-fermion vertex with coupling G_V is replaced by the q-q-V vertex plus a vector mass term, as shown in Fig. 6.

At the one quark-loop level the couplings H_1, L_{10}, f_V and the mass M_V get NTL logarithmic corrections as we have shown in Sect. 3.3.

Because of the presence of independent unknown coupling constants the running of the two quantities $f_V^2/2$ and $2H_1 + L_{10}$ is not *a priori* the same. There are two possible solutions at $Q^2 \neq 0$:

- The running with Q^2 of the two parameters can be different, while their values at $Q^2 = 0$ are related through the identity (88). In this case the coefficients β^i_V and β^1_Γ of the NTL logarithmic corrections are not constrained.
- The relation (88) has to be scale invariant. This puts a constraint on the coefficients of the NTL logarithmic corrections to $f_V^2/2$ and $2H_1 + L_{10}$, β^i_V and β^1_Γ.

The second solution appears to hold in resonance models and under the saturation hypothesis formulated in [6]. For kinematical reasons the CV model is the only vector model

which does not generate the saturation of the L_i, H_i counterterms of the $\mathscr{L}_4$ Lagrangian through vector resonance exchange. In the ENJL model the saturation is replaced by the direct contribution of one loop of quarks. Other vector models [6] saturate the relation (88) without the inclusion of quark-loops contribution. By construction the saturation by resonance exchange holds at the resonance scale ($Q^2 = M_V^2$). If we require a) the equivalence of the vector models (including the quark-loops contribution in the case of the CV model) and b) the validity of the saturation hypothesis, which in fact is experimentally well verified, we conclude that the relation (88) has to be scale invariant.
Let us see if this ansätz is satisfied by the coefficients $\beta^i_{V,\Gamma}$. The values of the two parameters of (88), including the NPLL corrections, can be deduced by using the formulas in Appendix A and B:

$$\begin{aligned}
\frac{f_V}{\sqrt{2}} &= \sqrt{Z_V} + \frac{N_c}{16\pi^2}\frac{1}{3}\frac{1}{\sqrt{Z_V}}\frac{Q^2}{\Lambda_\chi^2}\left[\frac{\beta^1_\Gamma}{2}\int_0^1 d\alpha P_1(\alpha)\ln\frac{\Lambda_\chi^2}{S(\alpha)}\right.\\
&\left.-\frac{1}{2}\beta^1_V\int_0^1 d\alpha P_1(\alpha)\ln\frac{\Lambda_\chi^2}{S(\alpha)}\right]\\
-(2H_1+L_{10}) &= Z_V^l + \frac{N_c}{16\pi^2}\frac{2}{3}\frac{Q^2}{\Lambda_\chi^2}\frac{\beta^1_\Gamma}{2}\int_0^1 d\alpha P_1(\alpha)\ln\frac{\Lambda_\chi^2}{S(\alpha)}\\
&= Z_V + \frac{N_c}{16\pi^2}\frac{1}{3}\frac{Q^2}{\Lambda_\chi^2}\left[2\frac{\beta^1_\Gamma}{2}\int_0^1 d\alpha P_1(\alpha)\ln\frac{\Lambda_\chi^2}{S(\alpha)}\right.\\
&\left.-\beta^1_V\int_0^1 d\alpha P_1(\alpha)\ln\frac{\Lambda_\chi^2}{S(\alpha)}\right],
\end{aligned}\tag{93}$$

where the wave function renormalization constant Z_V has been calculated in (75).

If we compare the running of the two terms of the relation (88) up to the NPLL order, we have:

$$\begin{aligned}
\frac{f_V^2}{2} &= Z_V + \frac{N_c}{16\pi^2}\frac{1}{3}\frac{Q^2}{\Lambda_\chi^2}\left[2\frac{\beta^1_\Gamma}{2}\int_0^1 d\alpha P_1(\alpha)\ln\frac{\Lambda_\chi^2}{S(\alpha)}\right.\\
&\left.-\beta^1_V\int_0^1 d\alpha P_1(\alpha)\ln\frac{\Lambda_\chi^2}{S(\alpha)}\right]\\
-(2H_1+L_{10}) &= Z_V + \frac{N_c}{16\pi^2}\frac{1}{3}\frac{Q^2}{\Lambda_\chi^2}\\
&\left[2\frac{\beta^1_\Gamma}{2}\int_0^1 d\alpha P_1(\alpha)\ln\frac{\Lambda_\chi^2}{S(\alpha)} - \beta^1_V\int_0^1 d\alpha P_1(\alpha)\ln\frac{\Lambda_\chi^2}{S(\alpha)}\right].
\end{aligned}\tag{94}$$

They have the same running in Q^2 including the NPLL corrections.

$\Pi^1_V(Q^2)$ can be written as follows:

$$\Pi^1_V(Q^2) = -4(2H_1+L_{10})(Q^2) - \frac{2f_V^2(Q^2)Q^2}{M_V^2(Q^2)+Q^2}. \tag{95}$$

By using the property that the running of the two parameters in (94) is the same (at least up to the NPLL order) the following expression holds:

$$\Pi^1_V(Q^2) = \frac{2f_V^2(Q^2)M_V^2(Q^2)}{M_V^2(Q^2)+Q^2}, \tag{96}$$

where the running of f_V^2 and M_V^2 is given by:

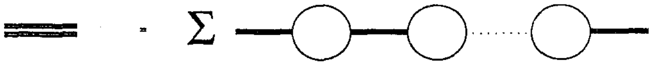

Fig. 7. The "dressed" vector meson propagator is given by the resummation of n quark-loop diagrams which are leading in the $1/N_c$ expansion

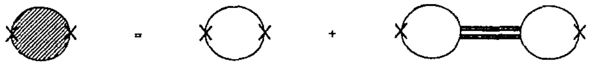

Fig. 8. The full vector two-point function as predicted by the QR model which we remind is developed at the leading order in the $1/N_c$ expansion. The vector meson propagator of the second term is defined in Fig. 7

$$\begin{aligned}
f_V^2 &= 2Z_V + \frac{N_c}{16\pi^2}\frac{2}{3}\frac{Q^2}{\Lambda_\chi^2}\left[2\frac{\beta^1_\Gamma}{2}\int_0^1 d\alpha P_1(\alpha)\ln\frac{\Lambda_\chi^2}{S(\alpha)}\right.\\
&\left.-\beta^1_V\int_0^1 d\alpha P_1(\alpha)\ln\frac{\Lambda_\chi^2}{S(\alpha)}\right]\\
M_V^2 &= \frac{N_c}{16\pi^2}\left(\frac{\Lambda_\chi^2}{2G_V}\right)\frac{1}{Z_V}\\
Z_V &= \frac{N_c}{16\pi^2}\frac{1}{3}\left[6\int_0^1 d\alpha\alpha(1-\alpha)\ln\frac{\Lambda_\chi^2}{S(\alpha)}\right.\\
&\left.+\beta^1_V\frac{Q^2}{\Lambda_\chi^2}\int_0^1 d\alpha P_1(\alpha)\ln\frac{\Lambda_\chi^2}{S(\alpha)}\right].
\end{aligned}\tag{97}$$

The infinite resummation of quark bubbles considered in [12] corresponds to replacing in the vector contribution the one quark-loop dressed propagator as shown in Fig. 7.

The set of diagrams corresponding to the full two-point vector correlation function predicted by the QR model is shown in Fig. 8.

4.2 *Determination of $\Pi^1_V(Q^2)$ at NTL order from experimental data*

The real part of the invariant Π function is related to its imaginary part through a standard dispersion relation

$$\mathrm{Re}\Pi^1_V(Q^2) = \int_0^\infty ds\frac{\frac{1}{\pi}\mathrm{Im}\Pi^1_V(s)}{s+Q^2}. \tag{98}$$

For a review on QCD spectral Sum rules and the calculation of QCD two-point Green's functions see [18].

For our analysis we choose the channel of the hadronic current with the ρ meson quantum numbers ($I=1, J=1$) $J^\rho_\mu = 1/\sqrt{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)$. The imaginary part of Π^1_V is experimentally known in terms of the total hadronic ratio of the e^+e^- annihilation in the isovector channel defined as follows:

$$R^{I=1}(s) = \frac{\sigma^{I=1}(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}. \tag{99}$$

The following dispersion relation holds [19, 20]:

$$\mathrm{Re}\Pi^1_V(Q^2) = \frac{2}{12\pi^2}\int_0^\infty ds\frac{R^{I=1}(s)}{s+Q^2}. \tag{100}$$

We have performed a comparison between the QR model parametrization of the vector 2-point function in the isovector channel, valid in the energy region $0 < Q^2 < \Lambda_\chi^2$, and the prediction obtained from a modelization of the experimental data on $e^+e^- \to hadrons$ [21]. For a determination of the function $\Pi_V^1(Q^2)$ in the high Q^2 region (i.e. beyond the cutoff Λ_χ) see [22].

We adopted the following parametrization of the experimental hadronic isovector ratio:

$$R^{I=1}(s) = \frac{9}{4\alpha^2} \frac{\Gamma_{ee}\Gamma_\rho}{(\sqrt{s} - m_\rho)^2 + \frac{\Gamma^2}{4}} + \frac{3}{2}\left(1 + \frac{\alpha_s(s)}{\pi}\right)\theta(s - s_0). \tag{101}$$

This is a generalization of the one proposed in [19], where the rho meson width corrections have not been included. $\Gamma_{ee} = 6.7 \pm 0.4$ KeV is the $\rho \to e^+e^-$ width and $\Gamma_\rho = 150.9 \pm 3.0$ is the total widht of the neutral ρ [23]. We used the leading logarithmic approximation for $\alpha_s(s)$:

$$\alpha_s(s) = \frac{12\pi}{33 - 2n_f} \frac{1}{\log(s/\Lambda_{\rm QCD}^2)}. \tag{102}$$

The modelization (101) includes a dependence of the ρ channel upon the ρ width and the contribution from the continuum starting at a threshold $s_0 = 1.5\, GeV^2$ [19]. For the running of α_s we used a value of 260 MeV for $\Lambda_{\rm QCD}$, according to the average experimental value $\Lambda_{\rm QCD}^{(4)} = 260^{+54}_{-46}$ MeV [23] and with $n_f = 4$ flavours.

The results are practically insensitive to the α_s running corrections and our leading log approximation turns out to be adequate.

The vector Green's function in the QR model has been parametrized in (96, 97). To extract information on $\beta_\Gamma^1, \beta_V^1$ coefficients of the NTL logarithmic corrections we made a best fit of the first derivative of the 2-point function coming from the modelization (101) of the experimental data:

$$\Pi'(Q^2)_{\rm exp} = -\frac{2}{12\pi^2}\int_0^\infty ds \frac{R^{I=1}(s)}{(s + Q^2)^2}, \tag{103}$$

where the derivative of the VV function in the QR model is given by:

$$\Pi'(Q^2)_{\rm QR} = \frac{\left[2f_V^{2\prime}\left(1 + \frac{Q^2}{M_V^2}\right) - 2\frac{f_V^2}{M_V^2}\left(1 - Q^2\frac{M_V^{2\prime}}{M_V^2}\right)\right]}{\left(1 + \frac{Q^2}{M_V^2}\right)^2}. \tag{104}$$

We have used $M_Q = 265$ MeV for the IR cutoff and $\Lambda_\chi = 1.165$ GeV for the UV cutoff, determined by a global fit in [8].

The fit has been done in the region: $0.5 < Q < 0.9$ GeV. Below the lower limit the NPLL corrections $Q^2/\Lambda_\chi^2 \ln(\Lambda_\chi^2/Q^2)$ are of the same order of the neglected corrections proportional to M_Q^2 and of order M_Q^2/Λ_χ^2 ln (Λ_χ^2/Q^2). Beyond the upper limit we are sufficiently near the cutoff to require the inclusion of higher order contributions.

In Fig. 9 we show the Q^2 behaviour of the derivative of the experimental 2-point function, the curve from the best fit, and the derivative of the ENJL prediction with quark-bubbles resummation of (91, 92). The best values of the two free coefficients are:

$$\beta_\Gamma^1 = -0.75 \pm 0.01 \quad \beta_V^1 = -0.79 \pm 0.01 \tag{105}$$

The χ^2 of the fit has been defined as $\sum_i(\Pi_i' - \Pi_i'^{exp})^2/\sigma_i^2$ and the σ_i are defined assuming a 10% of uncertainty on the experimental data. A $\chi^2/n.d.f. = 0.2$ has been obtained. The type of corrections we have analyzed are not the only ones. Apart from higher order corrections in the $1/\Lambda_\chi$ expansion, possible next-to-leading corrections in the $1/N_c$ expansion can be present. The ENJL prediction differs by roughly a 40% from the experimental curve at 0.8 GeV. Most of this discrepancy can be accounted for with the corrections that we have calculated.

The invariant function $\Pi_V^1(Q^2)$ obtained from the best fit automatically match the ENJL function at $Q = M_Q$, because we have normalized the corrections to vanish at $Q^2 = M_Q^2$:

$$\Pi_V^1(Q^2) = \Pi_V^{\rm ENJL}(Q^2)\theta(M_Q^2 - Q^2) + \int_{M_Q^2}^{Q^2} \frac{d\Pi_V^{\rm Fit}}{dQ'^2} dQ'^2 \theta(Q^2 - M_Q^2). \tag{106}$$

The $\Pi_V^1(Q^2)$ function obtained with the values (105) and with the matching of (106) is plotted in Fig. 10 and compared with the ENJL prediction of (91) (i.e. including the resummation of linear chains of quark bubbles and including only logarithmic corrections). The difference between the two curves reaches a 30% at 0.7 GeV.

The inclusion of gluons in the ENJL model makes worse the agreement with the experimental data.

The modelization of (101) does not include the higher $I = 1, J = 1$ resonance states with ρ quantum numbers $\rho(1450), \rho(1700)$. There is no measurement at present of their leptonic width. The addition of more resonance states

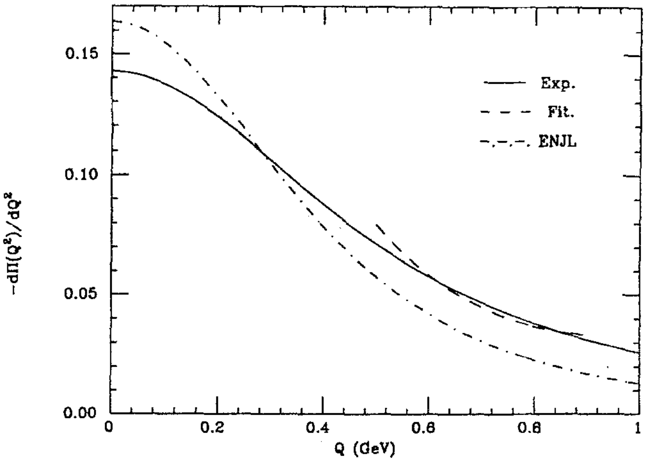

Fig. 9. The derivative of the experimental vector-vector function $-d\Pi_V^1(Q^2)/dQ^2$ (solid line), the fitted curve of the QR model (dashed line) and the prediction of the ENJL model including quark-bubble resummation and the logarithmic contributions in the incomplete Gamma functions $\Gamma(0, x)$ [12] (dot-dashed line) are shown as a function of $\sqrt{Q^2}$. The fit has been performed in the region $0.5 \geq \sqrt{Q^2} \leq 0.9$ GeV

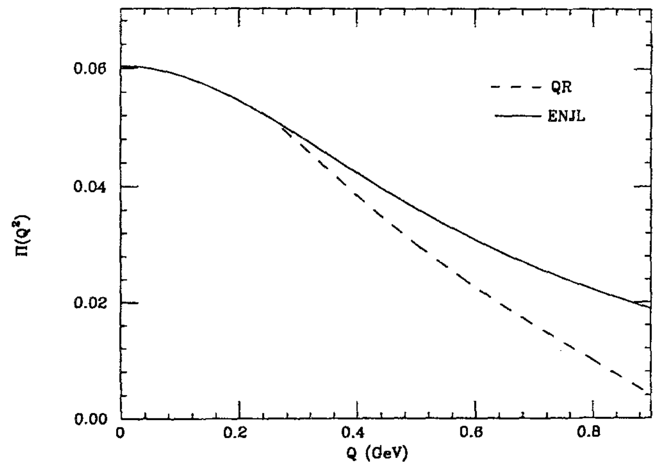

Fig. 10. The invariant function $\Pi_V^1(Q^2)$ (dashed line) is obtained from the fitted derivative of Fig. 9 by imposing the matching with the ENJL function at $Q = M_Q$. The ENJL prediction of (91 (full line) is also shown. Gluon contributions have not been included

increases the difference between the two curves. The sensitivity to the continuum threshold value s_0 of $R^{I=1}(s)$ is contained inside a 10% of variation in the range $s_0 = 1.5 \div 4\, GeV^2$. The practical insensitivity to large variations of the $\Lambda_{\rm QCD}$ parameter, due to the smallness of the contributions involving α_s, has been also verified.

The analysis of the vector-vector Green's function shows how a sizable magnitude of NPLL corrections can be estimated from the data. Correlations in other channels which are experimentally less accessible could be estimated by QCD lattice simulations which could be used to fix the parameters of the effective Lagrangian.

5 Conclusions

Effective quark models inspired to the old Nambu-Jona Lasinio model [9] have proven to be a promising tool to describe low energy hadronic interactions. In this type of models the hadron fields are introduced through the bosonization of the effective quark action. The effective meson Lagrangian comes from the integration over the quarks and gluons degrees of freedom.

The simplest model that one can construct is the so called ENJL model [8], where only the lowest dimension effective quark operators are included, leading in the $1/\Lambda_\chi$ and $1/N_c$ expansions.

As we have shown in detail, the ENJL model correctly predicts the value of the parameters of the effective meson Lagrangian in the zero energy limit. In this limit the model is noticeably more predictive with respect to the usual effective meson Lagrangian approach [1, 2, 6, 7]. As an example, the twelve counterterms of the effective pseudoscalar meson Lagrangian at order p^4 in the chiral expansion together with the parameters of the chiral leading effective resonance Lagrangian are all expressed in terms of only three input parameters of the NJL model: G_S, G_V and Λ_χ. Adding gluon corrections to order $\alpha_s N_c$ introduces ten more unknown constants which can be estimated in terms of a single unkown parameter g [8].

Nevertheless, the ENJL model is not able to describe the behaviour of the low energy hadronic observables at $Q^2 \neq 0$.

We indicate a systematic way to get predictions on the behaviour of the hadronic observables in the whole low energy range of Q^2 (i.e. $0 < Q^2 < \Lambda_\chi^2$) which could provide a bridge between the non-asymptotic and the asymptoptic regime of QCD.

The Quark-Resonance model formulated in this work is based on the inclusion of higher dimension n-quark effective interactions which modify the values of the low energy hadronic observables at $Q^2 \neq 0$.

Higher dimension operators produce next-to-leading power - leading log corrections of the type $(Q^2/\Lambda_\chi^2)\ln(\Lambda_\chi^2/Q^2)$ to the parameters of the effective meson Lagrangian and corrections without logarithms of order (Q^2/Λ_χ^2). The former are produced by a finite set of $1/\Lambda_\chi^2$ terms, while the latter arise from an infinite tower of higher dimension operators.

We have focused our attention on the first class of contributions, which are assumed to be dominant for values of Q^2 above the IR cutoff M_Q^2 and below the UV cutoff Λ_χ^2.

We have shown explicitely how the next-to-leading power - leading log corrections enter the calculation of the two-point vector Green's function. In the $I = 1, J = 1$ channel we were able to fix the four coefficients of these corrections through a fit to the experimental data on the $e^+e^- \to$ *hadrons* cross section. The comparison with the ENJL prediction of [12] provides evidence for a quantitative relevance of the next-to-leading terms in the $1/\Lambda_\chi$ expansion in the Q^2 dependence of the hadronic observables throughout the intermediate Q^2 region.

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Appendix A. Effective potential calculation: $n = 2$

The expression of a generic contribution at n=2 in Euclidean space is the following:

$$\frac{1}{2}\int\int dxdy\mathrm{Tr}\int\frac{d^4k}{(2\pi)^4}e^{ik(x-y)}\frac{1}{i\hat{k}+M_Q}\delta(y)$$
$$\int\frac{d^4q}{(2\pi)^4}e^{iq(y-x)}\frac{1}{i\hat{q}+M_Q}\delta(x), \tag{107}$$

where Tr is the trace over Dirac, colour and flavour indices.

It corresponds to a quark-loop diagram with two insertions of the operator $\delta(x)$ as defined in (57 and 55).

Defining $l \equiv k - q$ and introducing the Feynman parameter α, the formula reduces to:

$$-\frac{1}{2}\int\int dxdy\int\frac{d^4l}{(2\pi)^4}e^{il(x-y)}\int_0^1 d\alpha\int\frac{d^4k'}{(2\pi)^4}$$
$$\frac{1}{[k'^2+\alpha(1-\alpha)l^2+M_Q^2]^2}.$$

$$
\mathrm{Tr}\Big\{[i(\hat{k}' + \alpha\hat{l}) - M_Q]\delta(y)[i(\hat{k}' - (1-\alpha)\hat{l})
-M_Q]\delta(x)\Big\}. \quad (108)
$$

We give here the final formula for the contributions diverging logarithmically with the cutoff Λ_χ obtained with the insertion of three different forms of the local operator $\delta(x)$. These are the only calculations needed to obtain the corrections to the parameters of the vector meson Lagrangian generated by the insertion of one next-to-leading vertex $1/\Lambda_\chi^2$ and one leading vertex.

Case 1. $\delta(y) = \gamma_\mu(\gamma_5)\delta^\mu(y) \quad \delta(x) = \gamma_\mu(\gamma_5)\delta^\mu(x)$

$$
\Gamma_{log} = -\frac{1}{2}\frac{N_c\pi^2}{(2\pi)^8}\int\int dxdy\int d^4l e^{il(x-y)}(l_\mu l_\nu - g_{\mu\nu}l^2)
tr[\delta^\mu(y)\delta^\nu(x)]8\int_0^1 d\alpha\alpha(1-\alpha)\ln\frac{\Lambda^2}{S(\alpha)}, \quad (109)
$$

where tr is the trace over the flavour indices of the $\delta(x)$ matrices and $S(\alpha) = M_Q^2 + \alpha(1-\alpha)l^2$. Expression (109) can be simplified to:

$$
\Gamma_{log} = \frac{1}{2}\frac{N_c\pi^2}{(2\pi)^4}\int dy\, tr\Big[\delta^\mu(y)(\partial_\mu\partial_\nu - g_{\mu\nu}\partial^2)\delta^\nu(y)\Big]
8\int_0^1 d\alpha\alpha(1-\alpha)\ln\frac{\Lambda^2}{S(\alpha)} \quad (110)
$$

Case 2. $\delta(y) = \gamma_\mu(\gamma_5)\delta^\mu(y) \quad \delta(x) = \gamma_\mu(\gamma_5)\delta^{\mu\lambda}(x)\,\overrightarrow{\partial}_\lambda$

$$
\Gamma_{log} = -\frac{1}{2}\frac{N_c\pi^2}{(2\pi)^8}\int\int dxdy\int d^4l
e^{il(x-y)}il_\lambda(l_\mu l_\nu - g_{\mu\nu}l^2)
\mathrm{tr}\Big[\delta^\mu(y)\delta^{\nu\lambda}(x)\Big]8\int_0^1 d\alpha\alpha^2(1-\alpha)\ln\frac{\Lambda^2}{S(\alpha)}
$$
$$
= -\frac{1}{2}\frac{N_c\pi^2}{(2\pi)^4}\int dy\, tr\Big[\delta^\mu(y)\partial_\lambda(\partial_\mu\partial_\nu - g_{\mu\nu}\partial^2)\delta^{\nu\lambda}(y)\Big]
8\int_0^1 d\alpha\alpha^2(1-\alpha)\ln\frac{\Lambda^2}{S(\alpha)}. \quad (111)
$$

Case 3.a. $\delta(y) = \gamma_\mu(\gamma_5)\delta^\mu(y) \quad \delta(x) = \gamma_\mu(\gamma_5)\delta_\lambda(x)\,\overrightarrow{\partial}_\mu\overrightarrow{\partial}_\lambda$

$$
\Gamma_{log} = -\frac{1}{2}\frac{N_c\pi^2}{(2\pi)^8}\int\int dxdy\int d^4l e^{il(x-y)}tr\Big[\delta^\mu(y)\delta^\lambda(x)\Big]
\int_0^1 d\alpha\ln\frac{\Lambda^2}{S(\alpha)}\cdot\Big\{l^4 g_{\mu\lambda}\Big[3\alpha^2(1-\alpha)^2 - 2\alpha^3(1-\alpha)\Big]
+l^2 l_\mu l_\lambda\Big[12\alpha^2(1-\alpha)^2 - 8\alpha^3(1-\alpha)\Big]\Big\}
$$
$$
= -\frac{1}{2}\frac{N_c\pi^2}{(2\pi)^4}\int dy\int_0^1 d\alpha\ln\frac{\Lambda^2}{S(\alpha)}tr\Big[\delta^\mu(y)\Big\{(\partial^2)^2 g_{\mu\lambda}
\Big[3\alpha^2(1-\alpha)^2 - 2\alpha^3(1-\alpha)\Big]
+\partial^2\partial_\mu\partial_\lambda\Big[12\alpha^2(1-\alpha)^2 - 8\alpha^3(1-\alpha)\Big]\Big\}\delta^\lambda(x)\Big] \quad (112)
$$

Case 3.b. $\delta(y) = \gamma_\mu(\gamma_5)\delta^\mu(y) \quad \delta(x) = \gamma_\mu(\gamma_5)\delta_\mu(x)\,\overrightarrow{\partial}^2$

$$
\Gamma_{log} = -\frac{1}{2}\frac{N_c\pi^2}{(2\pi)^8}\int\int dxdy\int d^4l e^{il(x-y)}tr\Big[\delta^\mu(y)\delta^\nu(x)\Big]
\int_0^1 d\alpha\ln\frac{\Lambda^2}{S(\alpha)}\cdot\Big\{l^4 g_{\mu\nu}\Big[-18\alpha^2(1-\alpha)^2 + 12\alpha^3(1-\alpha)\Big]
+l^2 l_\mu l_\nu\Big[24\alpha^2(1-\alpha)^2 - 16\alpha^3(1-\alpha)\Big]\Big\}
$$
$$
= -\frac{1}{2}\frac{N_c\pi^2}{(2\pi)^4}\int dy\int_0^1 d\alpha\ln\frac{\Lambda^2}{S(\alpha)}tr\Big[\delta^\mu(y)\Big\{(\partial^2)^2 g_{\mu\nu}
[-18\alpha^2(1-\alpha)^2 + 12\alpha^3(1-\alpha)\Big]
+\partial^2\partial_\mu\partial_\nu\Big[24\alpha^2(1-\alpha)^2 - 16\alpha^3(1-\alpha)\Big]\Big\}\delta^\nu(y)\Big]. \quad (113)
$$

We have not included logarithmic terms proportional to the IR cutoff mass M_Q.

Appendix B. Effective potential calculation: $n = 3$

The expression of a generic contribution at n=3 in Euclidean space is the following:

$$
-\frac{1}{3}\int\int\int dxdydz\, Tr\int\frac{d^4k}{2\pi^4}e^{ik(x-y)}\frac{1}{i\hat{k} + M_Q}\delta(y)
\int\frac{d^4r}{2\pi^4}e^{ir(y-z)}\frac{1}{i\hat{r} + M_Q}\cdot\delta(z)\int\frac{d^4q}{2\pi^4}e^{iq(z-x)}\frac{1}{i\hat{q} + M_Q}\delta(x). \quad (114)
$$

By defining $l \equiv k - q$ and $m \equiv r - q$ and by introducing the Feynman parameters α, β the integral reduces to:

$$
-\frac{2}{3}\int\int\int dxdydz\int\int\frac{d^4l}{2\pi^4}\frac{d^4m}{2\pi^4}e^{il(x-y)+im(y-z)}
\int_0^1 d\alpha\int_0^{1-\alpha} d\beta\int\frac{d^4k'}{2\pi^4}\mathrm{Tr}
\frac{[(i(\hat{k}'+\hat{l}_1) - M_Q)\delta(y)(i(\hat{k}'+\hat{l}_2) - M_Q)\delta(z)(i(\hat{k}'+\hat{l}_3) - M_Q)\delta(x)]}{[k'^2 + S(\alpha,\beta)]^3}, \quad (115)
$$

where

$$
\begin{aligned}
l_1 &\equiv -\alpha(m - l) + \beta l\\
l_2 &\equiv (1-\alpha)(m - l) + \beta l\\
l_3 &\equiv -\alpha(m - l) - (1-\beta)l
\end{aligned} \quad (116)
$$

and

$$
S(\alpha,\beta) \equiv \alpha(1-\alpha)(m - l)^2
+\beta(1-\beta)l^2 + 2\alpha\beta l(m - l) + M_Q^2. \quad (117)
$$

The case which enters in the calculation of the NPLL corrections to the parameters of the two-point vector Green's function corresponds to the insertion of the following local δ operators:

$$\delta(y) = \gamma_\mu \delta^\mu(y) \quad \delta(z) = \gamma_\nu \delta^\nu(z)$$
$$\delta(x) = \frac{1}{\Lambda_\chi^2}\gamma_\lambda\{\overrightarrow{\partial}^\lambda, \overrightarrow{\partial}^2\}. \tag{118}$$

Formula (115) simplifies to:

$$-\frac{2}{3}\int\int\int dxdydz \int\int \frac{d^4l}{2\pi^4}\frac{d^4m}{2\pi^4} e^{il(x-y)+im(y-z)}$$
$$L_{\mu\nu}(m,l)\, tr[\delta_\mu(y)\delta_\nu(z)]$$
$$= -\frac{2}{3}\int\int dydz \int \frac{d^4m}{2\pi^4} e^{im(y-z)}$$
$$L_{\mu\nu}(m, l=0)\, tr[\delta_\mu(y)\delta_\nu(z)], \tag{119}$$

where $L_{\mu\nu}(m, l=0)$ is given by:

$$L_{\mu\nu}(m, l=0) = N_c \int_0^1 d\alpha \int_0^{1-\alpha} d\beta \int \frac{d^4k'}{2\pi^4}\frac{-i}{[k'^2 + S(\alpha)]^3}$$
$$\mathrm{Tr}[(i(\hat{k}' - \alpha\hat{m}) - M_Q)\gamma_\mu(i(\hat{k}' + (1-\alpha)\hat{m})$$
$$-M_Q)\gamma_\nu(i(\hat{k}' - \alpha\hat{m}) - M_Q)\gamma_\lambda](k' - \alpha m)^\lambda(k' - \alpha m)^2. \tag{120}$$

The logarithmically divergent contribution with the exclusion of terms proportional to the IR cutoff mass M_Q is given by:

$$L_{\mu\nu}(m, l=0) = -\frac{4\pi^2}{(2\pi)^4}N_c\int_0^1 d\alpha$$
$$\left\{ g_{\mu\nu}m^4[3\alpha^3(1-\alpha)^2 - \frac{3}{2}\alpha^4(1-\alpha)] + m_\mu m_\nu m^2 \right.$$
$$\left. [-4\alpha^3(1-\alpha)^2 + 2\alpha^4(1-\alpha)] \right\} \ln\frac{\Lambda_\chi^2}{S(\alpha)}, \tag{121}$$

where again $S(\alpha) = \alpha(1-\alpha)m^2 + M_Q^2$.

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